SENSITIVITY ANALYSIS OF SOLUTION MAPPINGS OF PARAMETRIC GENERALIZED QUASI VECTOR EQUILIBRIUM PROBLEMS

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Abstract. In this talk, we study the parametric generalized quasi vector equilibrium problem (PGQVEP). We investigate existence of solution for PGQVEP and continuities of the solution mappings of PGQVEP. In particular, results concerning the lower semicontinuity of the solution mapping of PGQVEP are presented.

1. INTRODUCTION

Let X be nonempty subset of a real topological vector space and $\mathbb{Z}$ a real topological vector space. A set $\mathcal{C} \subset \mathbb{Z}$ is said to be a cone if $\lambda x \in \mathcal{C}$ for any $\lambda \geq 0$ and for any $x \in \mathcal{C}$. The cone $\mathcal{C}$ is called proper if it is not whole space, i.e., $\mathcal{C} \neq \mathbb{Z}$. A cone $\mathcal{C}$ is said to be solid if it has nonempty interior, i.e., $\operatorname{int} \mathcal{C} \neq \emptyset$. Let $C : \mathcal{X} \to 2^{\mathbb{Z}}$ which has proper convex cone values. For any set $A \subset \mathbb{Z}$, we let $\operatorname{bd} A$ and $\operatorname{cl} A$ denote the boundary and closure of A, respectively. Also, we denote A^c the complement of the set A. For any set A of a real vector space, the convex hull of A, denoted by $\operatorname{co} A$, is the smallest convex set containing A. Furthermore, we denote zero vector of $\mathbb{Z}$ by $\theta_{\mathbb{Z}}$.

Let $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ and $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. For fixed $p \in \mathbb{P}$, the parametric generalized quasi vector equilibrium problem (PGQVEP) is to find $x \in K(p, x)$ such that

$$\text{(PGQVEP)} \qquad F(p, x, y) \not\subset -\operatorname{int} C(p, x), \text{ for all } y \in K(p, x).$$

Let $\Omega : \mathbb{P} \to 2^X$ be the set-valued mapping such that $\Omega(p)$ is the solutions set of PGQVEP for $p \in \mathbb{P}$, i.e.,

$$\Omega(p) = \{x \in K(p, x) : F(p, x, y) \not\subset -\operatorname{int} C(p, x), \text{ for all } y \in K(p, x)\}.$$

For fixed $p \in \mathbb{P}$, the parametric extended quasi vector equilibrium problem (PEQVEP) is to find $x \in K(p, x)$ such that

$$\text{(PEQVEP)} \quad F(p, x, y) \cap (-\operatorname{int} C(p, x)) = \emptyset, \text{ for all } y \in K(p, x).$$

Let $\Phi : \mathbb{P} \to 2^X$ be the set-valued mapping such that $\Phi(p)$ is the solutions set of PEQVEP for $p \in \mathbb{P}$, i.e.,

$$\Phi(p) = \{x \in K(p, x) : F(p, x, y) \cap (-\operatorname{int} C(p, x)) = \emptyset, \ \forall y \in K(p, x)\}.$$

In the literature, existence results for a (generalized) vector quasi equilibrium problems has been investigated. See, e.g., [3, 9]. If for each fixed $p \in \mathbb{P}$, K and C have constant values for every $x \in X$, respectively, PGQVEP reduce to a parametric vector equilibrium problem (PVEP). Existence of solution and closedness of solution mapping for PVEP has been studied in [6]. Continuity of solution mapping for PVEP has been studied in [7].

We observe that our results in this paper can be employed to study the behavior of solution maps of parametric vector optimization, parametric vector variational inequalities, parametric vector equilibrium problems and those generalized problems and so on.

2. Preliminaries

Definition 2.1 (*C*-continuity, [8])**.** Let X be a topological space and $\mathbb{Z}$ a topological vector space with a partial ordering defined by a proper solid convex cone C. Suppose that f is a vector-valued function from X to $\mathbb{Z}$. Then, f is said to be *C-continuous at* $x \in X$, if for any neighbourhood $V_{f(x)} \subset \mathbb{Z}$ of $f(x)$, there exists a neighbourhood $U_x \subset X$ of x such that $f(u) \in V_{f(x)} + C$ for all $u \in U_x$. Moreover a vector-valued function f is said to be *C-continuous in* X if f is C-continuous at every x on X.

Definition 2.2 (Continuity for Set-valued mapping, See also [1])**.** Let X and Y be two topological spaces, $T : X \to 2^Y$ a set-valued mapping.

(i) T is said to be *upper semicontinuous* (u.s.c. for short) at $x \in X$ if for each open set V containing $T(x)$, there is an open set U containing x such that for each $z \in U$, $T(z) \subset V$; T is said to be u.s.c. on X if it is u.s.c. at all $x \in X$.

(ii) T is said to be *lower semicontinuous* (l.s.c. for short) at $x \in X$ if for each open set V with $T(x) \cap V \neq \emptyset$, there is an open set U containing x such that for each $z \in U$, $T(z) \cap V \neq \emptyset$; T is said to be l.s.c. on X if it is l.s.c. at all $x \in X$.

(iii) T is said to be *continuous* at $x \in X$ if $T(x)$ is both u.s.c. and l.s.c.; T is said to be *continuous* on X if it is both u.s.c. and l.s.c. at each $x \in X$.

Proposition 2.1. *Let X be a topological space and $\mathbb{Z}$ a real topological vector space. Suppose that $C : X \to 2^{\mathbb{Z}}$ has proper solid convex cone values and that $W : X \to 2^{\mathbb{Z}}$ is defined by $Z \setminus (-\operatorname{int} C(x))$. Then we have the following two statements:*

(i) *if C is u.s.c. at x, then there exists a neighborhood U of x such that*

$$\operatorname{cl} C(x) \supset C(u), \ \ for \ all \ u \in U;$$

(ii) *if W is u.s.c. at x, then there exists a neighborhood U of x such that*

$$W(x) \supset W(u), \ \ for \ all \ u \in U.$$

Definition 2.3. Let X and $\mathbb{Z}$ be two real vector spaces. Suppose that K is a nonempty convex set of X and that $T : X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$.

(i) T is said to be convex mapping on K if for each $x_1, \ x_2 \in K$ and $\mu \in [0, 1]$

$$T(\mu x_1 + (1 - \mu)x_2) \supset \mu T(x_1) + (1 - \mu)T(x_2);$$

(ii) T is said to be concave mapping on K if for each $x_1,\ x_2 \in K$

and $\mu \in [0, 1]$

$$T(\mu x_1 + (1 - \mu)x_2) \subset \mu T(x_1) + (1 - \mu)T(x_2);$$

(iii) T is said to be affine on K if T is convex and concave mapping

on K.

Definition 2.4 (C-compactness). [8] Let C be a nonempty convex

cone in a Hausdorff topological space Z. We say $E \subset Z$ is C-compact

if any cover of E of the form

$$\{\mathcal{U}_\alpha + C : \alpha \in I,\ \mathcal{U}_\alpha \text{ are open}\}$$

admits a finite subcover.

Definition 2.5 (C-semicontinuity, See also [8]). Let X and Z be a

topological space and a real topological vector space, respectively. Let

$T : X \to 2^Z \setminus \{\emptyset\}$ and $C : X \to 2^Z$, which has proper convex cone

values. Let $x' \in X$.

(i) T is said to be $C(x)$-*lower semicontinuous* ($C(x)$-l.s.c.) at x if

for each $\mathcal{V}$, an open set of Z with $T(x) \cap \mathcal{V} \neq \emptyset$, there exists a

neighborhood $\mathcal{U}$ of x such that

$$T(u) \cap \big(\mathcal{V} + \operatorname{int} C(x)\big) \neq \emptyset,\ \text{for all } u \in \mathcal{U}.$$

T is said to be C-*lower semicontinuous* (C-l.s.c.) on X if T is

$C(x)$-l.s.c. at x for every $x \in Z$.

(ii) T is said to be $C(x)$-*upper semicontinuous* ($C(x)$-u.s.c.) at x,

if for each neighborhood $\mathcal{V}_{T(x)}$ of $T(x)$ there exists a neighbor-

hood $\mathcal{U}_x$ of x such that

$$T(u) \subset \mathcal{V}_{T(x)} + \operatorname{int} C(x), \ \text{for all } u \in \mathcal{U}_x.$$

T is said to be C-*upper semicontinuous* (C-u.s.c.) on X if T

is $C(x)$-u.s.c. at x for every $x \in Z$.

Definition 2.6 (Generalized C-quasiconvexity)**.** Let X be a vector

space, and Z also a vector space with a proper solid convex cone C.

Suppose that K is a convex subset of X and that $T : K \to 2^Z \setminus \{\emptyset\}$.

Then T is said to be *generalized C-quasiconvex on K* if for each $z \in Z$,

$$A(z) := \{x \in K : T(x) \subset z - C\}$$

is convex or empty.

Definition 2.7 (Extended C-quasiconvexity)**.** Let X be a vector space,

and Z also a vector space with a proper solid convex cone C. Suppose

that K is a convex subset of X and that $T : K \to 2^Z \setminus \{\emptyset\}$. Then T is

said to be *extended C-quasiconvex on K* if for each $z \in Z$,

$$A(z) := \{x \in K : T(x) \cap (z - C) \neq \emptyset\}$$

is convex or empty.

Definition 2.8 (C-quasiconcavity)**.** [13] Let X be a nonempty convex subset of a real topological vector space and Z a real topological vector space. Let $T : X \rightarrow 2^Z \setminus \{\emptyset\}$. Suppose that $C : X \rightarrow 2^Z$ has proper solid convex cone values. We say that T is *C-quasiconcave* on X if for each $x_1, x_2 \in X$ and $z \in Z$, $T(x_1) \not\subset z - \operatorname{int} C(x_1)$ and $T(x_2) \not\subset z - \operatorname{int} C(x_2)$ imply

$$T(x_\mu) \not\subset z - \operatorname{int} C(x_\mu), \text{ for all } x_\mu \in (x_1, x_2).$$

We also say that T is *strictly C-quasiconcave* on X if for each $x_1, x_2 \in X$ and $z \in Z$, $T(x_1) \not\subset z - \operatorname{int} C(x_1)$ and $T(x_2) \not\subset z - \operatorname{int} C(x_2)$ imply

$$T(x_\mu) \not\subset z - \operatorname{cl} C(x_\mu), \text{ for all } x_\mu \in (x_1, x_2).$$

Definition 2.9 (C-proper quasiconcavity)**.** [13] Let X be a nonempty convex subset of a real topological vector space and Z a real topological vector space. Let $T : X \rightarrow 2^Z \setminus \{\emptyset\}$. Suppose that $C : X \rightarrow 2^Z$ has proper solid convex cone values. We say that T is *C-proper quasiconcave* on X if for each $x_1, x_2 \in X$ and $z \in Z$, $T(x_1) \cap \big(z - \operatorname{int} C(x_1)\big) =$

$\emptyset$ and $T(x_2) \cap \big(z - \operatorname{int} C(x_2)\big) = \emptyset$ imply

$$T(x_\mu) \cap \big(z - \operatorname{int} C(x_\mu)\big) = \emptyset, \text{ for all } x_\mu \in (x_1, x_2).$$

We also say that T is *strictly C-properly quasiconcave* on X if for each $x_1, x_2 \in X$ and $z \in Z$, $T(x_1) \cap \big(z - \operatorname{int} C(x_1)\big) = \emptyset$ and $T(x_2) \cap \big(z - \operatorname{int} C(x_2)\big) = \emptyset$ imply

$$T(x_\mu) \cap \big(z - \operatorname{cl} C(x_\mu)\big) = \emptyset, \text{ for all } x_\mu \in (x_1, x_2).$$

Remark 1. If T is single-valued and C has constant values, Definitions 2.8 and 2.9 reduce to the definition of $(-C)$-proper quasiconvexity.

Definition 2.10 (C-weak quasiconcavity). Let X be a nonempty convex subset of a real topological vector space, Z a real topological vector space and $C : X \to 2^{\mathbb{Z}}$ with a proper solid convex cone values. Suppose $T : X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. We say that T is *C-weakly quasiconcave* on X if for each $x_1, x_2 \in X$, $T(x_1) \not\subset -\operatorname{int} C(x_1)$ and $T(x_2) \not\subset -\operatorname{int} C(x_2)$ imply

$$T(x_\mu) \not\subset -\operatorname{int} C(x_\mu), \text{ for all } x_\mu \in (x_1, x_2)$$

and also $T(x_1) \not\subset -\operatorname{int} C(x_1)$ and $T(x_2) \not\subset -\operatorname{cl} C(x_2)$ imply

$$T(x_\mu) \not\subset -\operatorname{cl} C(x_\mu), \text{ for all } x_\mu \in (x_1, x_2).$$

We say that T is *strictly C-weakly quasiconcave* on X if for each $x_1, x_2 \in$

X , $T(x_1) \not\subset -\text{int}\, C(x_1)$ and $T(x_2) \not\subset -\text{int}\, C(x_2)$ imply

$$T(x_\mu) \not\subset -\text{cl}\, C(x_\mu), \ \text{for all } x_\mu \in (x_1, x_2).$$

Example 1. Let $X = [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}]$ and $\mathbb{Z} = \mathbb{R}^2$. Let

$$C(x, y) = \{(u, v) \in \mathbb{Z} : u \cos x + v \sin x \geq 0\}.$$

Suppose

$$T(x, y) = (x - \frac{\pi}{4})(\sin x, -\cos x) + \{(y - x - t)(\cos x, \sin x) : t \in [0, 1]\}.$$

Then T is C-weakly quasiconcave on X.

Definition 2.11 (C-weak proper quasiconcavity)**.** Let X be a nonempty

convex subset of a real topological vector space, $\mathbb{Z}$ a real topological

vector space and $C : X \to 2^{\mathbb{Z}}$ with a proper solid convex cone val-

ues. Suppose $T : X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. We say that T is *C-weakly properly*

quasiconcave on X if for each $x_1, x_2 \in X$, $T(x_1) \cap (z - \text{int}\, C(x_1)) \neq$

$\emptyset$ and $T(x_2) \cap (z - \text{int}\, C(x_2)) \neq \emptyset$ imply

$$T(x_\mu) \cap (z - \text{int}\, C(x_\mu)) \neq \emptyset, \ \text{for all } x_\mu \in (x_1, x_2)$$

and also $T(x_1) \cap (z - \operatorname{int} C(x_1)) \neq \emptyset$ and $T(x_2) \cap (z - \operatorname{cl} C(x_2)) \neq \emptyset$

imply

$$T(x_\mu) \cap (z - \operatorname{cl} C(x_\mu)) \neq \emptyset, \text{ for all } x_\mu \in (x_1, x_2).$$

We say that T is *strictly C-weakly properly quasiconcave* on X if for each

$x_1, x_2 \in X$, $T(x_1) \cap (z - \operatorname{int} C(x_1)) \neq \emptyset$ and $T(x_2) \cap (z - \operatorname{int} C(x_2)) \neq \emptyset$

imply

$$T(x_\mu) \cap (z - \operatorname{cl} C(x_\mu)) \neq \emptyset, \text{ for all } x_\mu \in (x_1, x_2).$$

Example 2. Let $X = [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}]$ and $\mathbb{Z} = \mathbb{R}^2$. Let

$$C(x, y) = \{(u, v) \in \mathbb{Z} : u \cos x + v \sin x \geq 0\}.$$

Suppose

$$T(x, y) = (x - \frac{\pi}{4})(\sin x, -\cos x) + \operatorname{co} \{(y - x)(\cos x, \sin x), (0, 0)\}.$$

Then T is C-weakly properly quasiconcave on X.

Definition 2.12 (C-diagonally quasiconcavity; see also [5])**.** Let X be a nonempty convex subset of a real vector space, $\mathbb{Z}$ a real vector space and $C : X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Suppose $T : X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$.

(i) T is said to be *Type I C-diagonally quasiconcave in its second argument*, if for any finite subset A of X and any $x \in \operatorname{co} A$, there exists $y \in A$ such that $T(x, y) \not\subset -\operatorname{int} C(x)$.

(ii) T is said to be *Type II C-diagonally quasiconcave in its second argument*, if for any finite subset A of X and any $x \in \operatorname{co} A$, there exists $y \in A$ such that $T(x, y) \cap (-\operatorname{int} C(x)) = \emptyset$.

Proposition 2.2. *Let X be a nonempty convex subset of a real vector space, $\mathbb{Z}$ a real vector space and $C : X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Suppose that $T : X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. We also assume the following two conditions:*

(i) *for each $x \in X$ $T(x, x) \not\subset -\operatorname{int} C(x)$;*

(ii) *for each $x \in X$ $T(x, \cdot)$ is generalized $C(x)$-quasiconvex on X.*

Then T is Type I C-diagonally quasiconcave in its second argument.

Proposition 2.3. *Let X be a nonempty convex subset of a real vector space, $\mathbb{Z}$ a real vector space and $C : X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Suppose that $T : X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. We also assume the following two conditions:*

(i) *for each $x \in X$ $T(x, x) \cap (-\operatorname{int} C(x)) = \emptyset$;*

(ii) *for each $x \in X$ $T(x, \cdot)$ is extended $C(x)$-quasiconvex on X.*

Then T is Type II C-diagonally quasiconcave in its second argument.

Definition 2.13 (Intersectional mapping [13])**.** Let X be a topological space and $\mathbb{Z}$ a nonempty set. Let $T, G : X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$, respectively. We say G is an *intersectional mapping* of T, if for each $x \in X$ there exist a neighborhood $\mathcal{U}_x$ of x such that

$$G(x) \subset \bigcap_{u \in \mathcal{U}_x} T(u).$$

Proposition 2.4. [13, Proposition 2.2] *Let X be a nonempty subset of a topological space and $\mathbb{Z}$ a real topological vector spaces, respectively. Let $C : X \to 2^{\mathbb{Z}}$, which has proper solid convex cone values. Supose that $W : X \to 2^{\mathbb{Z}}$ defined by*

$$\mathcal{W}(x) = \mathbb{Z} \setminus \operatorname{int} C(x)$$

has closed graph. Then C has at least one intersectional mapping, which has solid convex cone values.

Proposition 2.5. *Let E be a nonempty subset of a topological space and $\mathbb{Z}$ a real topological vector space. Let $C : E \to 2^{\mathbb{Z}}$ with proper solid convex cone values and $D : E \to 2^{\mathbb{Z}}$ an intersectional mapping*

of C with solid convex cone values. Suppose $F : E \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ and

$W : E \to 2^{\mathbb{Z}}$, defined by $W(x) = \mathbb{Z} \setminus \big(-\operatorname{int} C(x)\big)$. We also assume the

following conditions:

    (i) W has closed graph;

    (ii) F is $(-D)$-u.s.c on X

    (iii) $F(x)$ is $(-D(x))$-compact for each $x \in X$.

Then the set $\mathfrak{S} = \{x \in E : F(x) \subset -\operatorname{int} C(x)\}$ is open.

Proposition 2.6. *Let E be a nonempty subset of a topological space*

and $\mathbb{Z}$ a real topological vector space. Let $C : E \to 2^{\mathbb{Z}}$ with proper

solid convex cone values and $D : E \to 2^{\mathbb{Z}}$ an intersectional mapping

of C with solid convex cone values. Suppose $F : E \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ and

$W : E \to 2^{\mathbb{Z}}$, defined by $W(x) = \mathbb{Z} \setminus \big(-\operatorname{int} C(x)\big)$. We also assume the

following conditions:

    (i) W *has closed graph;*

    (ii) F *is $(-D)$-l.s.c on X.*

Then the set $\mathfrak{T} = \{x \in E : F(x) \cap (-\operatorname{int} C(x)) \neq \emptyset\}$ is open.

Definition 2.14. Let X be a topological space and Y an nonemptyset.

A set-valued map $F : X \to 2^Y$ is said to *have open lower sections*, if

the set $F^{-1}(y) = \{x \in X : y \in F(x)\}$ is open in X for every $y \in Y$.

Lemma 2.1. [11] *Let X be a topological space and Y a convex set of*

a real topological vector space. Let $F, G : X \to 2^Y$ be two set-valued

maps with open lower sections. Then:

    (i) *the set-valued map $H : X \to 2^Y$, defined by $H(x) = \operatorname{co}\big(F(x)\big)$*

        for all $x \in X$, has open lower sections;

    (ii) *the set-valued map $J : X \to 2^Y$, defined by $J(x) = F(x) \cap G(x)$*

        for all $x \in X$, has open lower sections.

Lemma 2.2. [10, Fan-Browder fixed-point theorem] *Let X be a nonempty*

compact convex subset of a real Hausdorff topological vector space. Sup-

pose that $F : X \to 2^X$ is a set-valued map with nonempty convex values

and open lower sections. Then F has a fixed point.

3. Existence of solution for PGQVEP

In this section we drive some existence results for PGQVEP.

Theorem 3.1. *Let X be a nonempty subset of a real topological vector space and $\mathbb{Z}$ a real topological vector space, $\mathbb{P}$ an index set and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

(i) *for each $p \in \mathbb{P}$ $F(p, \cdot, \cdot)$ is Type I C-diagonally quasiconcave in the third argument;*

(ii) *X is compact and convex;*

(iii) *K has convex values and has open lower sections;*

(iv) *for each fixed $p \in \mathbb{P}$ and $x \in X$, the set*

$$\{y \in X : F(p, x, y) \subset -\operatorname{int} C(p, x)\}$$

is open.

Then Ω is nonempty for each $p \in \mathbb{P}$. Moreover, Φ is nonempty for each $p \in \mathbb{P}$ if conditions (i) *and* (iv) *are replaced by the following* (v) *and* (vi):

(v) *for each $p \in \mathbb{P}$ $F(p, \cdot, \cdot)$ is Type II C-diagonally quasiconcave*

in the third argument;

(vi) *the set $\{y \in X : F(p, x, y) \cap (-\operatorname{int} C(p, x)) \neq \emptyset\}$ is open, for*

each $p \in \mathbb{P}$ and $x \in X$.

Next we concider sufficient condition for assumptions (iv) and (vi) of Theorem 3.1.

Proposition 3.1. *Let E be a nonempty subset of a real topological vector space and $\mathbb{Z}$ a real topological vector space with a proper solid convex cone C Suppose $F : E \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. If F is $(-C)$-u.s.c. on E, then the set $\{x \in E : F(x) \subset -\operatorname{int} C\}$ is open. If F is $(-C)$-l.s.c. on E, then the set $\{x \in E : F(x) \cap (-\operatorname{int} C) \neq \emptyset\}$ is open.*

To investigate upper and lower semicontinuities of Ω and Φ, we need to require closedness of $K(p, x)$ for each $p \in \mathbb{P}$ and $x \in X$. The following theorem is useful.

Theorem 3.2. *Let X be a nonempty subset of a real topological vector space and $\mathbb{Z}$ a real topological vector space, $\mathbb{P}$ an index set and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to$*

$2^X \setminus \{\emptyset\}$. *Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume*

that the following conditions:

    (i) $\Omega(p)$ *is nonempty for each $p \in \mathbb{P}$;*

    (ii) *the set $\{y \in X : F(p, x, y) \subset -\operatorname{int} C(p, x)\}$ is open for each*

        fixed $p \in \mathbb{P}$ and $x \in X$.

Then

$$\overline{\Omega}(p) = \{x \in \operatorname{cl} K(p, x) : F(p, x, y) \not\subset -\operatorname{int} C(p, x), \ \text{for all } y \in \operatorname{cl} K(p, x)\}$$

is nonempty for each $p \in \mathbb{P}$.

Theorem 3.3. *Let X be a nonempty subset of a real topological vector*

space and $\mathbb{Z}$ a real topological vector space, $\mathbb{P}$ an index set and $C :$

$\mathbb{P} \times X \to 2^{\mathbb{Z}}$ *with proper solid convex cone values. Let $K : \mathbb{P} \times X \to$*

$2^X \setminus \{\emptyset\}$. *Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume*

that the following conditions:

    (i) $\Psi(p)$ *is nonempty for each $p \in \mathbb{P}$;*

    (ii) *the set $\{y \in X : F(p, x, y) \cap (-\operatorname{int} C(p, x)) \neq \emptyset\}$ is open for*

        each fixed $p \in \mathbb{P}$ and $x \in X$.

Then

$$\overline{\Psi}(p) = \{x \in \operatorname{cl} K(p, x) : F(p, x, y) \cap (-\operatorname{int} C(p, x)) = \emptyset, \ \forall y \in \operatorname{cl} K(p, x)\}$$

is nonempty for each $p \in \mathbb{P}$.

The following result is a consequence of Theorems 3.1, 3.2 and 3.3.

Theorem 3.4. *Let X be a nonempty subset of a real topological vector space and $\mathbb{Z}$ a real topological vector space, $\mathbb{P}$ an index set and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K, K' : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

(i) *for each $p \in \mathbb{P}$ $F(p, \cdot, \cdot)$ is Type I C-diagonally quasiconcave in the third argument;*

(ii) *X is compact and convex;*

(iii) *K' has convex values and open lower sections;*

(iv) *$K(p, x) = \operatorname{cl} K'(p, x)$ for each $p \in \mathbb{P}$ and $x \in X$;*

(v) *the set $\{y \in X : F(p, x, y) \subset -\operatorname{int} C(p, x)\}$ is open, for each fixed $p \in \mathbb{P}$ and $x \in X$.*

Then Ω is nonempty for each $p \in \mathbb{P}$. Moreover, Φ is nonempty for each $p \in \mathbb{P}$ if conditions (i) and (v) are replaced by the following (vi) and (vii):

(vi) *for each $p \in \mathbb{P}$ $F(p, \cdot, \cdot)$ is Type II C-diagonally quasiconcave in the third argument;*

(vii) *the set $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \cap (-\operatorname{int} C(p, x)) \neq \emptyset\}$ is open.*

Remark 2. Conditions (iii) and (iv) and the condition that K has closed convex values and open lower sections are quite different. For example let $\mathbb{P} = \{1\}$, $X = [0, 1]$, $A = (0, \frac{1}{2})$ and $B = (\frac{1}{2}, 1)$. Suppose that $K : \mathbb{P} \times X \to 2^X$ is defined by

$$K(p, x) = \operatorname{cl}\big(xA + (1 - x)B\big).$$

Then K satisfies conditions (iii) and (iv) but K does not have open lower sections.

Let

$$\Omega'(p) := \{x \in K(p, x) : F(p, x, y) \not\subset -\operatorname{cl} C(p, x), \ \forall y \in K(p, x)\}$$

and

$$\Phi'(p) := \{x \in K(p, x) : F(p, x, y) \cap (-\operatorname{cl} C(p, x)) = \emptyset, \ \forall y \in K(p, x)\}.$$

Theorem 3.5. *Let X be a nonempty subset of a real topological vector space and $\mathbb{Z}$ a real topological vector space, $\mathbb{P}$ an index set and $C : \mathbb{P} \times$*

$X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \backslash \{\emptyset\}$

and $c : \mathbb{P} \times X \to \mathbb{Z}$ with $c(p, x) \in \operatorname{int} C(p, x)$ for each $p \in \mathbb{P}$ and $x \in X$.

Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \backslash \{\emptyset\}$. Also we assume that the

following conditions:

(i) for each $p \in \mathbb{P}$, $F(p, \cdot, \cdot) - c(p, \cdot)$ is Type I C-diagonally qua-

siconcave in the third argument;

(ii) X is compact and convex;

(iii) K has convex values and open lower sections;

(iv) the set $\{y \in X : F(p, x, y) - c(p, x) \subset -\operatorname{int} C(p, x)\}$ is open,

for each fixed $p \in \mathbb{P}$ and $x \in X$.

Then $\Omega'(p)$ is nonempty for each $p \in \mathbb{P}$. Moreover, $\Phi'(p)$ is nonempty

for each $p \in \mathbb{P}$ if conditions (i) and (iv) are replaced by the following

(v) and (vi):

(v) for each $p \in \mathbb{P}$ $F(p, \cdot, \cdot) - c(p, x)$ is Type II C-diagonally qua-

siconcave in the third argument;

(vi) the set $\{y \in X : (F(p, x, y) - c(p, x)) \cap (-\operatorname{int} C(p, x)) \neq \emptyset\}$ is

open, for each $p \in \mathbb{P}$ and $x \in X$.

Theorem 3.6. *Let X be a nonempty subset of a real topological vector*

space and $\mathbb{Z}$ a real topological vector space, $\mathbb{P}$ an index set and $C :

$\mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K, K' : \mathbb{P} \times X \to$ $2^X \setminus \{\emptyset\}$ and $c : \mathbb{P} \times X \to \mathbb{Z}$ with $c(p, x) \in \operatorname{int} C(p, x)$ for each $p \in \mathbb{P}$ and $x \in X$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:

(i) for each $p \in \mathbb{P}$ $F(p, \cdot, \cdot) - c(p, \cdot)$ is Type I C-diagonally quasi-concave in the third argument;

(ii) X is compact and convex;

(iii) K' has convex values and open lower sections;

(iv) $K(p, x) = \operatorname{cl} K'(p, x)$ for each $p \in \mathbb{P}$ and $x \in X$;

(v) the set $\{y \in X : F(p, x, y) - c(p, x) \subset -\operatorname{int} C(p, x)\}$ is open for each fixed $p \in \mathbb{P}$ and $x \in X$.

Then Ω' is nonempty for each $p \in \mathbb{P}$. Moreover, Φ' is nonempty for each $p \in \mathbb{P}$ if conditions (i) and (iv) are replaced by the following (v) and (vi):

(vi) for each $p \in \mathbb{P}$ $F(p, \cdot, \cdot) - c(p, x)$ is Type II C-diagonally qua-siconcave in the third argument;

(vii) the set $\{y \in X : (F(p, x, y) - c(p, x)) \cap (-\operatorname{int} C(p, x)) \neq \emptyset\}$ is open, for each $p \in \mathbb{P}$ and $x \in X$.

4. Upper semicontinuity of the solution mapping

In this section we show that the solution mappings Ω of PGQVEP and Φ of PEQVEP are upper semicontinuous on $\mathbb{P}$, respectively, under suitable assumptions.

Theorem 4.1. *Let X be a nonempty subset of a real topological vector space and $\mathbb{Z}$ a real topological vector space, $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values and $\mathbb{P}$ a topological space. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

    (i) *$\Omega(p)$ is nonempty for each $p \in \mathbb{P}$;*

    (ii) *X is compact;*

    (iii) *$K(p, x)$ is compact for each $p \in \mathbb{P}$ and $x \in X$;*

    (iv) *K is u.s.c. on $\mathbb{P} \times X$;*

    (v) *the set $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \subset -\operatorname{int} C(p, x)\}$ is open.*

Then Ω is u.s.c. on $\mathbb{P}$. Moreover, Φ is u.s.c. on $\mathbb{P}$ if condition (v) is replaced by the following one:

    (vi) *the set $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \cap (-\operatorname{int} C(p, x)) \neq \emptyset\}$ is*

        open.

Theorem 4.2. *Let X be a nonempty subset of a real topological vector space and $\mathbb{Z}$ a real topological vector space, $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values and $\mathbb{P}$ a topological space. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

    (i) $\Omega(p)$ *is nonempty for each $p \in \mathbb{P}$;*

    (ii) X *is compact;*

    (iii) $K(p, x)$ *is compact for each $p \in \mathbb{P}$ and $x \in X$;*

    (iv) K *is u.s.c. on $\mathbb{P} \times X$;*

    (v) $W : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ *defined by $W(p, x) = \mathbb{Z} \setminus \left(-\operatorname{int} C(p, x) \right)$ is u.s.c. on $\mathbb{P} \times X$;*

    (vi) F *is $\left(-C \right)$-u.s.c. on $\mathbb{P} \times X \times X$.*

Then Ω is u.s.c. on $\mathbb{P}$. Moreover, Φ is u.s.c. on $\mathbb{P}$ if condition (vi) *is replaced by the following one:*

    (vii) F *is $\left(-C \right)$-l.s.c. on $\mathbb{P} \times X \times X$.*

Example 3. Let $\mathbb{P} = [0, 1]$, $\mathbb{X} = \mathbb{R}$, $X = [0, \frac{\pi}{2}]$, $Z = \mathbb{R}^2$, $A = (0, \frac{\pi}{4})$

and $B = (\frac{\pi}{4}, \frac{\pi}{2})$. Let

$$K(p, x) = \text{cl} \left(p\frac{2x}{\pi}A + \frac{2(1-x)}{\pi}B \right),$$

$$C(p, x) = \begin{cases} \{(u, v) \in Z : u \geq 0\}, & x \in [0, \frac{\pi}{6}) \text{ and } p \in [0, \frac{1}{2}), \\ \{(u, v) \in Z : u \geq 0 \text{ and } v \geq 0\}, & x \in [\frac{\pi}{6}, \frac{\pi}{3}] \text{ or } p \in [\frac{1}{2}, 1], \\ \{(u, v) \in Z : v \geq 0\}, & x \in (\frac{\pi}{3}, 1]) \text{ and } p \in [0, \frac{1}{2}). \end{cases}$$

Suppose that

$$F(p, x, y) = \begin{cases} \text{co}\left\{\begin{pmatrix} -p \\ -p \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right\}, & \text{if } x \leq y, x \in [0, \frac{\pi}{6}) \text{ and } p \in [0, \frac{1}{2}), \\ \text{co}\left\{\begin{pmatrix} -p \\ -p \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right\}, & \text{if } x \leq y, x \in [\frac{\pi}{6}, \frac{\pi}{3}] \text{ or } p \in [\frac{1}{2}, 1], \\ \text{co}\left\{\begin{pmatrix} -p \\ -p \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right\}, & \text{if } x \leq y, x \in (\frac{\pi}{3}, 1] \text{ and } p \in [0, \frac{1}{2}), \\ \left\{\begin{pmatrix} -p \\ -p \end{pmatrix}\right\}, & \text{otherwise.} \end{cases}$$

Then for each $p \in \mathbb{P}$, $\Omega(p) \neq \emptyset$. We also observe that every condition

of Theorem 4.2 is satisfied. Accordingly Ω is u.s.c. on $\mathbb{P}$. Indeed,

$$\Omega(p) = \begin{cases} \text{co}\{\frac{\pi}{6}, \frac{\pi}{4}\}, & p = 0, \\ \{\frac{\pi}{6}\}, & p \in (0, 1), \\ \text{co}\{\frac{\pi}{6}, \frac{\pi}{3}\}, & p = 1. \end{cases}$$

The following result is a consequence of Theorem 4.1 and Proposition 2.5.

Theorem 4.3. *Let X be a nonempty subset of a real topological vector*

space $\mathbb{X}$ and $\mathbb{Z}$ a real topological vector space, $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with

proper solid convex cone values and $\mathbb{P}$ a topological space. Let K :

$\mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. *Suppose that* $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ *is a*

vector-valued function and that $D : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ *is an intersectional*

mapping of C with solid convex cone values. Also we assume that the

following conditions:

(i) $\Omega(p)$ *is nonempty for each* $p \in \mathbb{P}$;

(ii) X *is compact;*

(iii) $K(p, x)$ *is compact for each* $p \in \mathbb{P}$ *and* $x \in X$;

(iv) K *is u.s.c. on* $\mathbb{P} \times X$;

(v) $W : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ *defined by* $W(p, x) = \mathbb{Z} \setminus \big(-\operatorname{int} C(p, x)\big)$ *has*

closed graph;

(vi) F *is* $\big(-D\big)$-*u.s.c on* $\mathbb{P} \times X \times X$;

(vii) $F(p, x, y)$ *is* $\big(-D(p, x)\big)$-*compact for each* $p \in \mathbb{P}$, $x \in X$ *and*

$y \in X$.

Then Ω is u.s.c. on $\mathbb{P}$.

The following result is a consequence of Theorem 4.1 and Proposition 2.6.

Theorem 4.4. *Let X be a nonempty subset of a real topological vector space $\mathbb{X}$ and $\mathbb{Z}$ a real topological vector space, $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values and $\mathbb{P}$ a topological space. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ is a vector-valued function and that $D : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ is an intersectional mapping of C with solid convex cone values. Also we assume that the following conditions:*

(i) *$\Phi(p)$ is nonempty for each $p \in \mathbb{P}$;*

(ii) *X is compact;*

(iii) *$K(p, x)$ is compact for each $p \in \mathbb{P}$ and $x \in X$;*

(iv) *K is u.s.c. on $\mathbb{P} \times X$;*

(v) *$W : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ defined by $W(p, x) = \mathbb{Z} \setminus \left(-\mathrm{int}\, C(p, x) \right)$ has closed graph;*

(vi) *F is $\left(-D \right)$-l.s.c on $\mathbb{P} \times X \times X$.*

Then Φ is u.s.c. on $\mathbb{P}$.

Example 4. Let $\mathbb{P}$, $\mathbb{X}$, X, $\mathbb{Z}$ and K be the same as those in Exmaple 3.

Let

$$C(p, x) = \left\{(u, v) \in Z : \left\langle \begin{pmatrix} \cos(x + p) \\ \sin(x + p) \end{pmatrix}, \begin{pmatrix} u \\ v \end{pmatrix} \right\rangle \geq 0 \right\},$$

$$D(p, x) = \left\{(u, v) \in Z : \begin{pmatrix} \cos x' & \sin x' \\ \cos x'' & \sin x'' \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \in \mathbb{R}_+^2 \right\},$$

where $x' = x + p - \dfrac{\pi}{32}$, $x'' = x + p + \dfrac{\pi}{32}$. Suppose

$$F(p, x, y) = \begin{cases} \mathrm{co}\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, (1 + (y - x)) \begin{pmatrix} \cos(x + p) \\ \sin(x + p) \end{pmatrix} \right\}, & \text{if } x \leq y, \\ \left\{ p \begin{pmatrix} -\cos(x + p) \\ -\sin(x + p) \end{pmatrix} \right\}, & \text{otherwise.} \end{cases}$$

Then for each $p \in \mathbb{P}$, $\Phi(p) \neq \emptyset$. We also observe that every condition

of Theorem 4.4 is satisfied. Accordingly S is u.s.c. on $\mathbb{P}$. Indeed

$$\Phi(p) = \begin{cases} \{\dfrac{\pi}{6}\} & p \in (0, 1], \\ [\dfrac{\pi}{6}, \dfrac{\pi}{4}] & p = 0. \end{cases}$$

5. Lower semicontinuity of the solution mapping

We next establish that the solution mappings Ω of PGQVEP and Φ of PEQVEP are lower semicontinuous on $\mathbb{P}$ under suitable assumptions.

Theorem 5.1. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

(i) *$\Omega'(p)$ is nonempty for each $p \in \mathbb{P}$;*

(ii) *$K(p, x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;*

(iii) *for each $p \in \mathbb{P}$ $K(p, \cdot)$ is affine on X;*

(iv) *K is u.s.c. on $\mathbb{P} \times X$;*

(v) *$\mathcal{F} : \mathbb{P} \to 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p, x)\}$, is l.s.c. on $\mathbb{P}$;*

(vi) *the set $\{(p, x, y) \in \mathbb{P} \times X \times X : F(p, x, y) \not\subset -\mathrm{cl}\, C(p, x) = \emptyset\}$ is open;*

(vii) *F is C-weakly quasiconcave on $\mathbb{P} \times X \times X$.*

Then Ω is l.s.c. on $\mathbb{P}$. Moreover, Φ is l.s.c. on $\mathbb{P}$ if conditions (vi) *and* (vii) *are replaced by the following* (viii) *and* (ix)

   (viii) *the set $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \cap (-\operatorname{cl} C(p, x)) \neq \emptyset\}$ is open;*

   (ix) *F is C-weakly properly quasiconcave on $\mathbb{P} \times X \times X$.*

Next we investigate condition (vi) and (viii).

Proposition 5.1. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

   (i) *F is $C(p, x)$-l.s.c. at (p, x, y) for every $p \in \mathbb{P}$, $x \in X$ and $y \in X$;*

   (ii) *C is u.s.c. on $\mathbb{P} \times X$.*

Then the set $\mathfrak{U} = \{(p, x, y) \in \mathbb{P} \times X \times X : F(p, x, y) \not\subset -\operatorname{cl} C(p, x)\}$ is open.

Proposition 5.2. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space*

and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K :$ $\mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:

(i) F is $C(p,x)$-u.s.c. at (p,x,y) for every $p \in \mathbb{P}$, $x \in X$ and $y \in X$;

(ii) C is u.s.c. on $\mathbb{P} \times X$.

Then the set $\mathfrak{U}' = \{(p,x,y) \in \mathbb{P} \times X \times X : F(p,x,y) \cap (-\mathrm{cl}\, C(p,x)) = \emptyset\}$ is open.

Proposition 5.3. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K :$ $\mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ and that $D : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ is a intersectional mapping of C with solid convex cone values. Also we assume that the following conditions:*

(i) *F is $D(p,x)$-l.s.c. at (p,x,y) for every $p \in \mathbb{P}$, $x \in X$ and $y \in X$;*

(ii) *C has closed graph.*

Then the set $\mathfrak{U} = \{(p,x,y) \in \mathbb{P} \times X \times X : F(p,x,y) \not\subset -\mathrm{cl}\, C(p,x)\}$ is open.

Proposition 5.4. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ and that $D : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ is a intersectional mapping of C with solid convex cone values. Also we assume that the following conditions:*

   (i) *F is $D(p, x)$-u.s.c. at (p, x, y) for every $p \in \mathbb{P}$, $x \in X$ and $y \in X$;*

   (ii) *$F(p, x, y)$ is $D(p, x)$-comapct for each $p \in \mathbb{P}$, $x \in X$ and $y \in X$;*

   (iii) *C has closed graph.*

Then the set $\mathfrak{A}' = \{(p, x, y) \in \mathbb{P} \times X \times X : F(p, x, y) \cap (-\mathrm{cl}\, C(p, x)) = \emptyset\}$ is open.

Theorem 5.2. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

   (i) *$\Omega'(p)$ is nonempty for each $p \in \mathbb{P}$;*

   (ii) *$K(p, x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;*

(iii) *for each $p \in \mathbb{P}$ $K(p, \cdot)$ is affine on X;*

(iv) *K is u.s.c. on $\mathbb{P} \times X$;*

(v) *$\mathcal{F} : \mathbb{P} \to 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p, x)\}$, is l.s.c. on $\mathbb{P}$;*

(vi) *F is $C(p, x)$-l.s.c. at (p, x, y) for every $p \in \mathbb{P}$, $x \in X$ and $y \in X$;*

(vii) *C is u.s.c. on $\mathbb{P} \times X$;*

(viii) *F is C-weakly quasiconcave on $\mathbb{P} \times X \times X$.*

Then Ω is l.s.c. on $\mathbb{P}$.

Theorem 5.3. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

(i) *$\Omega'(p)$ is nonempty for each $p \in \mathbb{P}$;*

(ii) *$K(p, x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;*

(iii) *for each $p \in \mathbb{P}$ $K(p, \cdot)$ is affine on X;*

(iv) *K is u.s.c. on $\mathbb{P} \times X$;*

(v) $\mathcal{F} : \mathbb{P} \to 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p,x)\}$, is

l.s.c. on $\mathbb{P}$;

(vi) F is $C(p,x)$-u.s.c. at (p,x,y) for every $p \in \mathbb{P}$, $x \in X$ and

$y \in X$;

(vii) C is u.s.c. on $\mathbb{P} \times X$;

(viii) F is C-weakly properly quasiconcave on $\mathbb{P} \times X \times X$.

Then Φ is l.s.c. on $\mathbb{P}$.

Theorem 5.4. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ and that $D : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ is a intersectional mapping of C with solid convex cone values. Also we assume that the following conditions:*

(i) $\Omega'(p)$ is nonempty for each $p \in \mathbb{P}$;

(ii) $K(p,x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;

(iii) for each $p \in \mathbb{P}$ $K(p,\cdot)$ is affine on X;

(iv) K is u.s.c. on $\mathbb{P} \times X$;

(v) $\mathcal{F} : \mathbb{P} \to 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p,x)\}$, is

l.s.c. on $\mathbb{P}$;

(vi) F *is* $D(p,x)$-*l.s.c. at* (p,x,y) *for every* $p \in \mathbb{P}$, $x \in X$ *and* $y \in X$;

(vii) C *has closed graph;*

(viii) F *is* C-*weakly quasiconcave on* $\mathbb{P} \times X \times X$.

Then Ω *is l.s.c. on* $\mathbb{P}$.

Example 5. Let $\mathbb{P}$, $\mathbb{X}$, X, K, $\mathbb{Z}$, C and D be the same as those in Example 4. Suppose

$$F'(p,x,y) = (x - \frac{\pi}{4})\begin{pmatrix} \sin(x+p) \\ -\cos(x+p) \end{pmatrix} + (y-x)\begin{pmatrix} \cos(x+p) \\ \sin(x+p) \end{pmatrix}$$

and

$$F(p,x,y) = \begin{cases} \mathrm{co}\left\{M(p), F'(p,x,y) - p(p-2)\begin{pmatrix} \cos(x+p) \\ \sin(x+p) \end{pmatrix}\right\} & \text{if } p < 1, \\ \{F'(p,x,y)\} & \text{otherwise,} \end{cases}$$

where

$$M(p) = \left\{ p\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} : \alpha \in [0, 2\pi) \right\}.$$

Then by Corollary **??**, we have

$$\Omega'(p) = \{x \in K(p,x) : F(p,x,y) \not\subset -\mathrm{cl}\, C(p,x), \text{ for all } y \in K(p,x)\} \neq \emptyset$$

for each $p \in \mathbb{P}$. Hence by Theorem 5.4, Ω is l.s.c. on $\mathbb{P}$. Indeed,

$$\Omega(p) = \begin{cases} \{x \in X : \frac{\pi}{6} \leq x \leq \frac{\pi}{6} + \frac{2}{3}p(2-p)\}, & \text{if } p \in [0,1), \\ \{x \in X : x = \frac{\pi}{6}\}, & \text{otherwise.} \end{cases}$$

Theorem 5.5. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ and that $D : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ is a intersectional mapping of C with solid convex cone values. Also we assume that the following conditions:*

(i) *$\Phi'(p)$ is nonempty for each $p \in \mathbb{P}$;*

(ii) *$K(p, x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;*

(iii) *for each $p \in \mathbb{P}$ $K(p, \cdot)$ is affine on X;*

(iv) *K is u.s.c. on $\mathbb{P} \times X$;*

(v) *$\mathcal{F} : \mathbb{P} \to 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p, x)\}$, is l.s.c. on $\mathbb{P}$;*

(vi) *F is $D(p, x)$-u.s.c. at (p, x, y) for every $p \in \mathbb{P}$, $x \in X$ and $y \in X$;*

(vii) *$F(p, x, y)$ is $D(p, x)$-compact for each $p \in \mathbb{P}$, $x \in X$ and $y \in X$;*

(viii) *C has closed graph;*

(ix) *F is C-weakly properly quasiconcave on $\mathbb{P} \times X \times X$.*

Then Φ is l.s.c. on $\mathbb{P}$.

Example 6. Let $\mathbb{P}$, $\mathbb{X}$, X, K, $\mathbb{Z}$, C, D and F' be the same as those in

Exmaple 5. Suppose

$$M(p) = \left\{ \begin{pmatrix} p \\ p \end{pmatrix} \right\}.$$

Then, Φ is l.s.c. on $\mathbb{P}$. Indeed,

$$\Phi(p) = \begin{cases} \left\{ x \in X : \dfrac{\pi}{6} \le x \le \dfrac{\pi}{6} + \dfrac{2}{3}p(2-p) \right\}, & \text{if } p \in [0,1), \\ \left\{ x \in X : x = \dfrac{\pi}{6} \right\}, & \text{otherwise.} \end{cases}$$

Theorem 5.6. *Let X and $\mathbb{P}$ be two nonempty subsets of two real*

topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space

and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let

$K : \mathbb{P} \times X \to 2^{X} \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$.

Also we assume that the following conditions:

(i) *$\Omega(p)$ has at least two elements for each $p \in \mathbb{P}$;*

(ii) *$K(p, x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;*

(iii) *for each $p \in \mathbb{P}$, $K(p, \cdot)$ is affine on X;*

(iv) *K is u.s.c. on $\mathbb{P} \times X$;*

(v) *$\mathcal{F} : \mathbb{P} \to 2^{X}$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p, x)\}$, is*

   l.s.c. on $\mathbb{P}$;

(vi) *the set $\{(p, x, y) \in \mathbb{P} \times X \times X : F(p, x, y) \not\subset -\text{cl}\, C(p, x)\}$ is*

   open;

(vii) F *is strictly C-weakly quasiconcave on $\mathbb{P} \times X \times X$.*

Then Ω is l.s.c. on $\mathbb{P}$.

Theorem 5.7. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

(i) *$\Phi(p)$ has at least two elements for each $p \in \mathbb{P}$;*

(ii) *$K(p, x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;*

(iii) *for each $p \in \mathbb{P}$, $K(p, \cdot)$ is affine on X;*

(iv) *K is u.s.c. on $\mathbb{P} \times X$;*

(v) *$\mathcal{F} : \mathbb{P} \to 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p, x)\}$, is l.s.c. on $\mathbb{P}$;*

(vi) *the set $\{(p, x, y) \in \mathbb{P} \times X \times X : F(p, x, y) \cap (-\operatorname{cl} C(p, x)) = \emptyset\}$ is open;*

(vii) *F is strictly C-weakly properly quasiconcave on $\mathbb{P} \times X \times X$.*

Then Φ is l.s.c. on $\mathbb{P}$.

Theorem 5.8. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \rightarrow 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \rightarrow 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \rightarrow 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

  (i) *$\Omega(p)$ has at least two elements for each $p \in \mathbb{P}$;*

  (ii) *$K(p, x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;*

  (iii) *for each $p \in \mathbb{P}$, $K(p, \cdot)$ is affine on X;*

  (iv) *K is u.s.c. on $\mathbb{P} \times X$;*

  (v) *$\mathcal{F} : \mathbb{P} \rightarrow 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p, x)\}$, is l.s.c. on $\mathbb{P}$;*

  (vi) *F is $C(p, x)$-l.s.c. at (p, x, y) for every $p \in \mathbb{P}$, $x \in X$ and $y \in X$;*

  (vii) *C is u.s.c. on $\mathbb{P} \times X$.*

  (viii) *F is strictly C-weakly quasiconcave on $\mathbb{P} \times X \times X$.*

Then Ω is l.s.c. on $\mathbb{P}$.

Theorem 5.9. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \rightarrow 2^{\mathbb{Z}}$ with proper solid convex cone values. Let*

$K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. *Suppose that* $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$.

Also we assume that the following conditions:

(i) $\Phi(p)$ *has at least two elements for each* $p \in \mathbb{P}$;

(ii) $K(p, x)$ *is convex and compact for each* $p \in \mathbb{P}$ *and* $x \in X$;

(iii) *for each* $p \in \mathbb{P}$, $K(p, \cdot)$ *is affine on* X;

(iv) K *is u.s.c. on* $\mathbb{P} \times X$;

(v) $\mathcal{F} : \mathbb{P} \to 2^X$, *defined by* $\mathcal{F}(p) = \{x \in X : x \in K(p, x)\}$, *is*

l.s.c. *on* $\mathbb{P}$;

(vi) F *is* $C(p, x)$-*u.s.c. at* (p, x, y) *for every* $p \in \mathbb{P}$, $x \in X$ *and*

$y \in X$;

(vii) $F(p, x, y)$ *is* $D(p, x)$-*compact for each* $p \in \mathbb{P}$, $x \in X$ *and* $y \in X$;

(viii) C *is u.s.c. on* $\mathbb{P} \times X$.

(ix) F *is strictly* C-*weakly properly quasiconcave on* $\mathbb{P} \times X \times X$.

Then Φ *is l.s.c. on* $\mathbb{P}$.

Theorem 5.10. *Let* X *and* $\mathbb{P}$ *be two nonempty subsets of two real topological vector spaces, respectively,* $\mathbb{Z}$ *a real topological vector space and* $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ *with proper solid convex cone values. Let* $K :$ $\mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. *Suppose that* $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ *and that*

$D : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ is a intersectional mapping of C with solid convex cone values. Also we assume that the following conditions:

    (i) $\Omega(p)$ has at least two elements for each $p \in \mathbb{P}$;

    (ii) $K(p,x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;

    (iii) for each $p \in \mathbb{P}$ $K(p, \cdot)$ is affine on X;

    (iv) K is u.s.c. on $\mathbb{P} \times X$;

    (v) $\mathcal{F} : \mathbb{P} \to 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p,x)\}$, is l.s.c. on $\mathbb{P}$;

    (vi) F is $D(p,x)$-l.s.c. at (p,x,y) for every $p \in \mathbb{P}$, $x \in X$ and $y \in X$;

    (vii) C has closed graph.

    (viii) F is C-weakly quasiconcave on $\mathbb{P} \times X \times X$.

Then Ω is l.s.c. on $\mathbb{P}$.

Theorem 5.11. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$ and that $D : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ is a intersectional mapping of C with solid convex cone values. Also we assume that the following conditions:*

(i) $\Phi(p)$ *has at least two elements for each* $p \in \mathbb{P}$;

(ii) $K(p,x)$ *is convex and compact for each* $p \in \mathbb{P}$ *and* $x \in X$;

(iii) *for each* $p \in \mathbb{P}$ $K(p,\cdot)$ *is affine on* X;

(iv) K *is u.s.c. on* $\mathbb{P} \times X$;

(v) $\mathcal{F} : \mathbb{P} \to 2^X$, *defined by* $\mathcal{F}(p) = \{x \in X : x \in K(p,x)\}$, *is*

    l.s.c. on $\mathbb{P}$;

(vi) F *is* $D(p,x)$-*u.s.c. at* (p,x,y) *for every* $p \in \mathbb{P}$, $x \in X$ *and*

    $y \in X$;

(vii) $F(p,x,y)$ *is* $D(p,x)$-*compact for each* $p \in \mathbb{P}$, $x \in X$ *and* $y \in X$;

(viii) C *has closed graph.*

(ix) F *is* C-*weakly properly quasiconcave on* $\mathbb{P} \times X \times X$.

Then Ω *is l.s.c. on* $\mathbb{P}$.

6. Continuity of the solution mapping

By combining results established in Sections 4 and 5, we have the following results concering continuity of the solution mappings Ω and Φ, respectively.

Theorem 6.1. *Let* X *and* $\mathbb{P}$ *be two nonempty subsets of two real topological vector spaces, respectively,* $\mathbb{Z}$ *a real topological vector space*

and $C : \mathbb{P} \times X \to 2^{\mathbb{Z}}$ with proper solid convex cone values. Let

$K : \mathbb{P} \times X \to 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \to 2^{\mathbb{Z}} \setminus \{\emptyset\}$.

Also we assume that the following conditions:

(i) $\Omega'(p)$ is nonempty for each $p \in \mathbb{P}$;

(ii) X is compact;

(iii) $K(p, x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;

(iv) for each $p \in \mathbb{P}$ $K(p, \cdot)$ is affine on X;

(v) K is u.s.c. on $\mathbb{P} \times X$;

(vi) $\mathcal{F} : \mathbb{P} \to 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p, x)\}$, is

l.s.c. on $\mathbb{P}$;

(vii) F is C-weakly quasiconcave on $\mathbb{P} \times X \times X$;

(viii) the set $\{(p, x, y) \in \mathbb{P} \times X \times X : F(p, x, y) \not\subset -\mathrm{cl}\, C(p, x) = \emptyset\}$

is open;

(ix) the set $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \subset -\mathrm{int}\, C(p, x)\}$ is open.

Then Ω is continuous on $\mathbb{P}$. Moreover, Φ is continuous on $\mathbb{P}$ if conditions (vii), (viii) and (ix) are replaced by the following (x), (xi) and (xii)

(x) F is C-weakly properly quasiconcave on $\mathbb{P} \times X \times X$;

(xi) *the set $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \cap (-\operatorname{cl} C(p, x)) \neq \emptyset\}$ is*

open;

(xii) *the set $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \cap (-\operatorname{int} C(p, x)) \neq \emptyset\}$ is*

open.

Theorem 6.2. *Let X and $\mathbb{P}$ be two nonempty subsets of two real topological vector spaces, respectively, $\mathbb{Z}$ a real topological vector space and $C : \mathbb{P} \times X \rightarrow 2^{\mathbb{Z}}$ with proper solid convex cone values. Let $K : \mathbb{P} \times X \rightarrow 2^X \setminus \{\emptyset\}$. Suppose that $F : \mathbb{P} \times X \times X \rightarrow 2^{\mathbb{Z}} \setminus \{\emptyset\}$. Also we assume that the following conditions:*

(i) *$\Omega(p)$ is nonempty for each $p \in \mathbb{P}$;*

(ii) *X is compact;*

(iii) *$K(p, x)$ is convex and compact for each $p \in \mathbb{P}$ and $x \in X$;*

(iv) *for each $p \in \mathbb{P}$ $K(p, \cdot)$ is affine on X;*

(v) *K is u.s.c. on $\mathbb{P} \times X$;*

(vi) *$\mathcal{F} : \mathbb{P} \rightarrow 2^X$, defined by $\mathcal{F}(p) = \{x \in X : x \in K(p, x)\}$, is*

l.s.c. on $\mathbb{P}$;

(vii) *F is strictly C-weakly quasiconcave on $\mathbb{P} \times X \times X$;*

(viii) *the set $\{(p, x, y) \in \mathbb{P} \times X \times X : F(p, x, y) \not\subset -\operatorname{cl} C(p, x) = \emptyset\}$*

is open;

(ix) *the set* $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \subset -\text{int}\, C(p, x)\}$ *is open.*

Then Ω is continuous on $\mathbb{P}$. Moreover, Φ is continuous on $\mathbb{P}$ if conditions (vii), (viii) and (ix) are replaced by the following (x), (xi) and (xii)

(x) *F is strictly C-weakly properly quasiconcave on $\mathbb{P} \times X \times X$;*

(xi) *the set* $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \cap (-\text{cl}\, C(p, x)) \neq \emptyset\}$ *is open;*

(xii) *the set* $\{(p, x, y) \in \mathbb{P} \times X : F(p, x, y) \cap (-\text{int}\, C(p, x)) \neq \emptyset\}$ *is open.*

Example 7. Let $\mathbb{P}$, $\mathbb{X}$, X, K, $\mathbb{Z}$, C, D and F' be the same as those in Example 5. Suppose

$$G(p, x, y) = \left\{ \begin{pmatrix} (x + p) \cos(x + p) \\ (y + p) \sin(y + p) \end{pmatrix} \right\}.$$

and

$$F(p, x, y) = \text{co}\, \{F'(p, x, y), G(p, x, y)\}.$$

Then Ω is continuous on $\mathbb{P}$. Indeed,

$$\Omega(p) = \Big\{ x \in X : \frac{\pi}{6} \leq x \leq \frac{\pi}{6} + \frac{2}{3}p(2 - p) \Big\}.$$

Example 8. Let $\mathbb{P}$, $\mathbb{X}$, X, K, $\mathbb{Z}$, C, D and G be the same as those in Example 7. Suppose

$$F(p, x, y) = \text{co} \left\{ F'(p, x, y), -(p^2 - 4) \begin{pmatrix} \cos(x + p) \\ \sin(x + p) \end{pmatrix} \right\}.$$

Then Ω is continuous on $\mathbb{P}$. Indeed,

$$\Phi(p) = \left\{ x \in X : \frac{\pi}{6} \le x \le \frac{\pi}{6} + \frac{2}{3}p(2 - p) \right\}.$$

7. ACKNOWLEDGEMENT

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