

# Nash Equilibrium and Second Order Games

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Nash equilibrium has inspired much research and applications in academic world. The primary concept is based on whoever deviates unilaterally from the equilibrium strategy cannot be better off. The concept offers only local stability, not global stability because players may move jointly to a better point is not considered. In addition, the fact that human psychology determines his/her action and behavior is also not fully incorporated. This presentation, based on previous works, shows how to incorporate human psychology in formulating games as people play them. In particular we formulate a two person game by the theory of Habitual Domains and the theory of Markov chains. Using Habitual Domains theory, we present a new model for describing the evolution of the mind sets of the players over time, the two person second order game. We introduce the solution concept of win-win mind profile, which offers global stability. Then, under some reasonable assumptions, we derive the Possibility Theorem that it is always possible to reach and maintain a win-win mind profile. In addition, we also provide a method to predict the average number of steps needed for a game to reach a win-win mind profile.

# Nash Equilibrium and Knowledge Sharing in Organization

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Knowledge is the most important resource in an organization, and the knowledge transfer and sharing between employees is of vital importance for organizations. “Prisoner’s dilemma” exists in the process of the organizational knowledge transfer and sharing when the employees transfer their knowledge to the organization and share their knowledge with other employees. This talk analyzes the process and obstacle of the knowledge transfer and sharing in the organization and studies the game model of the knowledge transfer and sharing. It suggests different knowledge potential employees should be stimulated by different measures. Through analyzing principle-agent in the incentive mechanism, the equity incentive method will have infinitely repeated games to the knowledge high-potential employees who are the key sources of the knowledge transfer and sharing in the incentive mechanism design. This makes it possible to break the prisoner’s dilemma of the knowledge transfer and sharing.

# Robust Nash Equilibrium Under Uncertainty

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We consider a stochastic generalized Nash equilibrium problem, where the players' cost functions involve uncertainty and nonsmoothness, and each player's strategy set depends on the rivals' strategies through shared stochastic variational inequality constraints. The expected residual minimization formulation is used to present a robust Nash equilibrium model with a nonempty and bounded solution set. The smoothing sample average approximation (SSAA) method is applied to solve the problem. Examples in oligopolistic transit market are presented to illustrate the problem and the robust Nash equilibrium model.

# Nash Equilibria of Differential Stochastic Game for Insurers with VaR Constraints

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We consider the stochastic differential game between two insurance companies with risk constraints. The companies are allowed to purchase proportional reinsurance and invest their money into both risk-free asset and risky assets. One company chooses the optimal reinsurance and investment strategy in order to maximize the expected payoff, and the other one tries to minimize this value. For the purpose of risk management, the risk arising from the whole portfolio is constrained to some level. The competition between the two companies is formulated as a two player stochastic differential game with constraints. By the dynamic programming principle, the problem is reduced to solving the Hamilton-Jacobi-Bellman-Isaacs (HJBI) equations for Nash equilibria. We derive the Nash equilibria explicitly and obtain the closed form solution to HJBI.

# Nash Equilibria of Dynamic Markov Zero-sum Game with Stopping

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We study the sufficient conditions for the existence of Nash equilibria of time-dependent discrete Markov zero-sum game up to a given stopping time. The stopping time is allowed to take either a finite or an infinite non-negative random variable with its associated objective function being well-defined. The result enables us to show the existence of the Nash equilibria of discrete games constructed by Markov chain approximation of a class of stochastic differential games.

# Generalized Nash Equilibrium

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Nash equilibrium in an N-person non-cooperative game is one of the most fundamental solution concepts in game theory. Recently, intensive efforts have been made, particularly in the optimization community, to extend the scope of the traditional non-cooperative games, or Nash equilibrium, in order to model complicated conflict situations that arise in practice. Notable such extended games include generalized Nash game, robust Nash game, multi-leader-follower game, and so on. This talk mainly focuses on the generalized Nash game and discusses its theoretical and numerical issues.