Plenary Talks

On the Pervasiveness of Difference-convexity in Optimization and Statistics

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Abstract. With the increasing interest in applying the methodology of difference-of-convex (dc) optimization to diverse problems in engineering and statistics, we show that many well-known functions arising therein can be represented as the difference of two convex functions. These include a univariate folded concave function commonly employed in statistical learning, the value function of a copositive recourse function in two-stage stochastic programming, and many composite statistical functions in risk analysis, such as the value-at-risk (VaR), conditional value-at-risk (CVaR), expectation-based, VaR-based, and CVaR-based random deviation functionals. We also discuss decomposition methods for computing directional stationary points of a class of nonsmooth, nonconvex dc programs that combined the Gauss-Seidel idea, the alternating direction method of multipliers, and a special technique to handle the negative of a pointwise max function.

This talk is based on joint work with Mahed Nouiehed, Meisam Razaviyayn, and Tao Min.

Data Based Nonlinear Distributions under Robust Expectations

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Abstract. A real world data sample is often modeled as an i.i.d. sequence, or as a linear and/or nonlinear regression model driven by an i.i.d. sequence. But in many situations this modeling is far from true. We must take into account the uncertainty essentially hidden inside a real world data sequence. We have to introduce a robust nonlinear expectation to quantitatively measure and calculate this type of uncertainty.

We have introduced a robust and simple algorithm of phi-max-mean which can be used to measure such type of uncertainties. In fact, it has provided an asymptotically optimal unbiased estimator to the corresponding nonlinear distribution.

Models for Neural Networks; Analysis, Simulations and Behaviour

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Abstract. Neurons exchange informations via discharges, propagated by membrane potential, which trigger firing of the many connected neurons. How to describe large networks of such neurons? What are the properties of these mean-field equations? How can such a network generate a spontaneous activity? Such questions can be tackled using nonlinear integro-differential equations. These are now classically used in the neuroscience community to describe neuronal networks or neural assemblies. Among them, the best known is certainly Wilson-Cowan's equation which describe spiking rates arising in different brain locations.

Another classical model is the integrate-and-fire equation that describes neurons through their voltage using a particular type of Fokker-Planck equations. Several mathematical results will be presented concerning existence, blow-up, convergence to steady state, for the excitatory and inhibitory neurons, with or without refractory states. Conditions for the transition to spontaneous activity (periodic solutions) will be discussed.

One can also describe directly the spike time distribution which seems to encode more directly the neuronal information. This leads to a structured population equation that describes at time t the probability to find a neuron with time s elapsed since its last discharge. Here, we can show that small or large connectivity leads to desynchronization. For intermediate regimes, sustained periodic activity occurs. A common mathematical tool is the use of the relative entropy method.

This talk is based on works with K. Pakdaman and D. Salort, M. Caceres, J. A. Carrillo and D. Smets.

Entropy Stable High Rrder Discontinuous Galerkin Methods for Hyperbolic Conservation Laws

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Abstract. It is well known that semi-discrete high order discontinuous Galerkin (DG) methods satisfy cell entropy inequalities for the square entropy for both scalar conservation laws and symmetric hyperbolic systems, in any space dimension and for any triangulations. However, this property holds only for the square entropy and the integrations in the DG methods must be exact. It is significantly more difficult to design DG methods to satisfy entropy inequalities for a non-square convex entropy, and / or when the integration is approximated by a numerical quadrature. In this talk, we report on our recent development of a unified framework for designing high order DG methods which will satisfy entropy inequalities for any given single convex entropy, through suitable numerical quadrature which is specific to this given entropy. Our framework applies from one-dimensional scalar cases all the way to multi-dimensional systems of conservation laws. For the one-dimensional case, our numerical quadrature is based on the methodology established in the literature, with the main ingredients being summation-by-parts (SBP) operators derived from Legendre Gauss-Lobatto quadrature, the entropy stable flux within elements, and the entropy stable flux at element interfaces. We then generalize the scheme to two-dimensional triangular meshes by constructing SBP operators on triangles based on a special quadrature rule. A local discontinuous Galerkin (LDG) type treatment is also incorporated to achieve the generalization to convection-diffusion equations. Numerical experiments will be reported to validate the accuracy and shock capturing efficacy of these entropy stable DG methods.

This is a joint work with Tianheng Chen.

Gaussian Mixtures and Their Tensors

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Abstract. Mixtures of Gaussians are ubiquitous in data science. We give an introduction to the geometry of these statistical models, with focus on the tensors that represent their higher moments. The familiar theory of rank and borderrank for symmetric tensors is recovered when all covariance matrices are zero. Recent work with Carlos Amendola and Kristian Ranestad characterizes the circumstances under which Gaussian mixtures are identifiable from their moments.

Statistical Methods for Measuring Mind

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Abstract. Measurement theory has played a foundational role in educational, psychological and psychiatric assessment. This talk will first discuss various statistical models that have been serving as key tools in measurement theory. It will then introduce new statistical models and discuss their theoretical properties and related algorithms. The new developments will be applied to examples in educational assessment and psychological evaluation.

Stream 1

Numerical Solutions of Differential Equations

Invited Talks

Stability of Implicit and Implicit–explicit Multistep Methods for Nonlinear Parabolic Equations in Hilbert Spaces

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Abstract. We consider the discretization of a class of nonlinear parabolic equations in Hilbert spaces by both implicit and implicit–explicit multistep methods, and establish local stability under best possible and best possible *linear* stability conditions, respectively. Our approach is based on a spectral and Fourier stability technique and uses an optimal quantification of the non-self-adjointness of linear elliptic operators as well as a discrete perturbation argument.

A Posteriori Error Estimators for Discontinuous Finite Element Approximations to Diffusion Problems

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Abstract. In this talk, we will describe both residual and equilibrated a posteriori error estimators for the nonconforming and the discontinuous Galerkin finite element approximations to diffusion problems. It is shown that both the residual and the equilibrated a posteriori error estimators are robust with respect to the diffusion coefficient, i.e., constants in the error bounds are independent of the jump of the diffusion coefficient. Moreover, we obtained some of those estimates with no assumption on the distribution of the diffusion coefficient.

Maximum-norm a Posteriori Error Analysis for Finite Element Methods

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Abstract. In this talk we give an overview of a posteriori error estimators for controlling finite element errors in maximum norms, discuss their analysis, and apply them to adaptive control of maximum errors in singularly perturbed reaction-diffusion equations. We also use the bounded mean oscillation norm as a proxy for the maximum norm in order to give some hints about possible pathways to obtaining convergence analysis for adaptive FEM for controlling maximum errors.

Fully Discrete Finite Element Methods for the Stochastic Allen-Cahn Equation with Gradient-type Multiplicative Noises

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Abstract. In this talk I shall present some recent developments on finite element approximations of the stochastic Allen-Cahn equation with gradient-type multiplicative noises and its sharp interface limit, the stochastic mean curvature flow (SMCF). The focuses of the talk will be on discussing the new difficulties arising from numerical analysis of this nonlinear stochastic PDE and on illustrating the analytical techniques for overcoming these difficulties. Numerical results obtained by fast Monte Carlo finite element solvers will also be presented.
This is a joint work with Yukun Li of Ohio State University and Yi Zhang of University of Notre Dame, U.S.A.

hp–Adaptive Newton-Discontinuous-Galerkin Finite Element Methods for Semilinear Elliptic Boundary Value Problems

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Abstract. In this talk we outline an *hp*–adaptive procedure for the numerical solution of general second-order semilinear elliptic boundary value problems, with possible singular perturbation. Our approach combines both adaptive Newton schemes and an *hp*–version adaptive discontinuous Galerkin finite element method (DGFEM). More precisely, this general technique is based on applying local Newton-type linearisations on the continuous level that allow for the approximation of the underlying semilinear PDE by a sequence of linearised problems. These resulting linear PDEs are then discretised by means of an *hp*–adaptive DGFEM procedure, based on employing standard residual–based *aposteriori* error bounds. The adaptive Newton-Galerkin procedure provides an interplay between the (adaptive, or damped) Newton method and the adaptive finite element approach, whereby we either perform a Newton step, if the Newton linearisation effect dominates, or enrich the current finite element space based on the *aposteriori* residual indicators, in the case that the finite element discretisation constitutes the main source of error. Numerical experiments are presented which highlight the robustness and reliability of the proposed approach for a variety of problems.

A Conservative Nonlocal Convection-Diffusion Model and Asymptotically Compatible Finite Difference Discretization

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Abstract. In this talk, we first propose and analyze a nonlocal convection-diffusion model, in which the convection term is constructed in a special upwind manner so that mass conservation and maximum principle are maintained. The well-posedness of the proposed nonlocal model and its convergence to the classical local convection-diffusion model are established. A quadrature-based finite difference discretization is then developed to numerically solve the nonlocal problem and it is shown to be consistent and unconditionally stable. We also demonstrate that the numerical scheme is asymptotically compatible, that is, the approximate solutions converge to the exact solution of the corresponding local problem when the nonlocal horizon parameter $\delta \to 0$ and the grid size $h \to 0$. Numerical experiments are performed to complement the theoretical analysis. (This work is done jointly with H. Tian from Ocean University of China and Q. Du from Columbia University, USA.)

Semi-discrete Finite Element Analysis of Time Fractional Diffusion Problems with Nonsmooth Initial Condition using Energy Arguments

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Abstract. In this talk, the conforming piecewise-linear finite element method (FEM) is applied to approximate the solution of time-fractional diffusion equations with variable diffusivity on bounded convex domains. Standard energy arguments do not provide satisfactory results for such a problem due to the low regularity of its exact solution. Using a delicate energy analysis, *a priori* optimal error bounds in L^2, H^1, and quasi-optimal in L^∞-norms are derived for the semidiscrete method for cases with smooth and nonsmooth initial data. The main tool of our analysis is based on a repeated use of an integral operator and use of a t^m type of weights to take care of the singular behavior of the continuous solution at $t = 0$. The generalized Leibniz formula for fractional derivatives is found to play a key role in our analysis. The present analysis can be extended to other types of fractional in time evolution problems.

A Priori Error Estimates of Parabolic Optimal Control Problems With Point Controls

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Abstract. We consider a parabolic optimal control problem with a pointwise control in space, but variable in time, in two or three space dimensions. To approximate the problem numerically we use the standard continuous piecewise linear approximation in space and the first order discontinuous Galerkin method in time. Despite low regularity of the state equation, we show almost optimal h^2+k convergence rate in 2D and $h+\sqrt{k}$ in 3D for the control in L^2 norm. I will explain the key regularity estimate and new a priori fully discrete global and local error estimates in $L^2([0,T];L^\infty(\Omega))$ norms for parabolic problems, that are essential in our analysis. If time permit I will also discuss optimal a priori error estimates of parabolic optimal control problems with a moving point control.

a New Anisotropic Finite Element Method on Polyhedral Domains

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Abstract. On a polyhedral domain $\Omega \subset \mathbb{R}^3$, consider the Poisson equation with the Dirichlet boundary condition. For singular solutions from the non-smoothness of the domain boundary, we propose new anisotropic mesh refinement algorithms to improve the convergence of finite element approximation. The proposed algorithm is simple, explicit, and requires less geometric conditions on the mesh and on the domain. Then, we develop interpolation error estimates in suitable weighted spaces for the anisotropic mesh, especially for the tetrahedra violating the maximum angle condition. These estimates can be used to design optimal finite element methods approximating singular solutions. We report numerical test results to validate the method.

Recent Advances on DG Methods for Maxwell's Equations in Complex Media

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Abstract. Inspired by the successful construction of metamaterials in 2000 and its many interesting potential applications (such as invisibility cloak and subwavelength imaging), we developed some DG methods for simulating wave propagation in metamaterials in recent years. In this talk, we will present a review of our past works on DG methods. More specifically, we like to talk about the posteriori error estimate obtained for the IPDG method developed for time-dependent integral-differential Maxwell's equations in cold plasma, and the edge element method for time-dependent Maxwell's equations. Application of a posteriori error estimator to invisibility cloak will be demonstrated. Some open issues will be mentioned.

Mass Conservative Domain Decomposition Methods for Multicomponent Contamination Flows

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Abstract. Due to the coupling and nonlinearity of multicomponent contamination equations, the long term computation and the large scale simulation, it is important to develop efficient domain decomposition methods for solving the multicomponent fluid flows in porous media and for applications of groundwater environment. Since non-overlapping domain decomposition methods have the advantages in the low computation and communication costs at each time step, some explicit-implicit non-overlapping domain decomposition methods have been developed in solving time dependent PDEs. However, these methods break the important physical law of mass conservation over multiple sub-domains. In this talk, we will present our recent development and analysis of the mass conservative domain decomposition FD schemes for parabolic equations and convection-diffusion equations in porous media. We will also show numerical simulations of the multicomponent contamination flows by using the developed conservative domain decomposition methods.

A-stable Time Discretizations Preserve Maximal Parabolic Regularity

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Abstract. Maximal L^p-regularity (for $1 < p < \infty$) is an important mathematical tool in the analysis of nonlinear parabolic partial differential equations. This talk addresses the question as to which time discretization methods preserve maximal L^p-regularity in the discrete ℓ^p-setting, uniformly in the stepsize. Moreover, the use of this concept in numerical analysis is illustrated.

It is shown that for a parabolic problem with maximal L^p-regularity, the time discretization by a linear multistep method or Runge–Kutta method has maximal ℓ^p-regularity uniformly in the stepsize if the method is A-stable (and satisfies minor additional conditions). In particular, the implicit Euler method, the Crank-Nicolson method, the second-order backward difference formula (BDF), and the Radau IIA and Gauss Runge–Kutta methods of all orders preserve maximal regularity. The proof uses Weis' characterization of maximal L^p-regularity in terms of R-boundedness of the resolvent, a discrete operator-valued Fourier multiplier theorem by Blunck, and generating function techniques that have been familiar in the stability analysis of time discretization methods since the work of Dahlquist. The A(α)-stable higher-order BDF methods have maximal ℓ^p-regularity under an R-boundedness condition in a larger sector.

As an illustration of the use of maximal regularity in the error analysis of discretized semilinear parabolic equations, it is shown how error bounds are obtained without using any growth condition on the nonlinearity or for nonlinearities having singularities. This becomes possible because discrete maximal regularity permits to control the maximum norm of the numerical solution and its gradient.

The talk is based on joint work with Balázs Kovács and Buyang Li, which appeared in SIAM J. Numer. Anal. 54 (2016), 3600-3624.

Convergence of the Immersed-boundary Finite-element Method for the Stokes Problem

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Abstract. The immersed boundary (IB) method is a powerful method for solving a class of fluid-structure interaction problems originally proposed by C. S. Peskin to simulate the blood flow through artificial heart valves. The IB method is also successfully applied to multi-phase flow problems, elliptic interface problems, and so on. In contrast to a huge number of applications, it seems that a few results are devoted to theoretical convergence analysis. In this paper, we report our recent study on the convergence of the IB method applied to a model Stokes problem with the homogeneous Dirichlet boundary condition. As a discretization method, we deal with the finite element method. First, the immersed force field is approximated using a regularized delta function and its error in the $W^{-1,p}$ norm is examined for $1 \leq p < n/(n-1)$, n being the space dimension. Then, we study the regularization and discretization errors separately. Consequently, error estimate of order $h^{1-\alpha}$ in the $W^{1,1} \times L^1$ norm for the velocity and pressure is derived, where α is an arbitrarily small positive number. Error estimate of order $h^{1-\alpha}$ in the L^r norm for the velocity is also derived with $r = n/(n-1-\alpha)$. The validity of those theoretical results are confirmed by numerical examples.
This is a joint work with Yoshiki Sugitani (AIMR, Tohoku University).

Recursive Integral Eigenvalue Solver with Cayley Transformation

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Abstract. Recently, a non-classical eigenvalue solver, called **RIM**, was proposed to compute (all) eigenvalues in a region on the complex plane. Without solving any eigenvalue problems, it tests a region to see if it contains eigenvalues using an approximate spectral projection. Regions that contain eigenvalues are subdivided and tested recursively until eigenvalues are isolated with a specified precision. This makes **RIM** an eigenvalue solver distinct from all existing methods. Furthermore, it requires no a priori spectral information. In this paper, we implement an improved version of **RIM** for non-Hermitian eigenvalue problems. Using Cayley transforms, the computation cost is reduced significantly also it inherits all the advantages of RIM. Numerical examples are presented and compared with 'eigs' in Matlab.

New Error Analysis of Fems for Incompressible Miscible Flows in Porous Media

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Abstract. In last several decades, numerous efforts have been made on numerical analysis for incompressible miscible flows in porous media. Optimal error estimates of various finite element methods seem well done. In this talk, we present our new work in this area. The traditional approach to optimal $L^\infty((0,T);L^2)$ error estimates is based on an elliptic Ritz projection, which usually requires the diffusion coefficient σ satisfying: $\nabla_x \partial_t \sigma \in L^p(\Omega_T)$. However, the commonly-used Bear–Scheidegger diffusion-dispersion tensor $D(\mathbf{u})$ in the incompressible miscible flows does not satisfy the regularity condition even for a smooth velocity field $\mathbf{u}$. Therefore, all the proofs in those previous works on optimal $L^\infty((0,T);L^2)$ error estimates may not be valid for Galerkin-Galerkin methods, Galerkin-mixed methods and many other numerical methods. A new approach is presented recently, in terms of a parabolic projection, which only requires the Lipschitz continuity of $D(\mathbf{u})$. With the new approach, we establish optimal L^p error estimates and an almost optimal L^∞ error estimate.

Novel Finite Element Methods for Elliptic Optimal Control Problems

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Abstract. In this talk we will present novel finite element methods for distributed elliptic optimal control problems with pointwise state and control constraints. These methods are based on the formulation of the optimal control problems as fourth order variational inequalities.

FERKN Integrators for Solving Nonlinear Wave Equations

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Abstract. The main theme of this talk is Extended Runge-Kutta-Nystrm integrators for solving nonlinear wave equations. We begin with ERKN integrators for multi-frequency highly oscillatory ODEs $y'' + My = f(y)$, with $\|M\| >> 1$, and then consider the error analysis for ERKN integrators for the multi-frequency highly oscillatory system, when applied to nonlinear wave equations. This leads to a new perspective for the ERKN integrators. That is, the error bound of ERKN methods is independent of $\|M\|$, when applied to nonlinear wave equations under certain assumptions. These assumptions appeared in the literature of error analysis for Gaustchi-type methods.

A New Nonlinear Finite Element Iterative Method for Solving a Nonuniform Ionic Size Modified Poisson-Boltzmann System

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Abstract. The nonuniform size-modified Poisson-Boltzmann equation (SMPBE) is a system of one second order elliptic interface boundary value problem and several nonlinear algebraic equations. Its solution can result in a prediction to an electrostatic potential function and ionic concentration functions for a protein in an ionic solvent. However, the development of efficient and effective numerical algorithms for solving SMPBE is very challenging due to strong solution singularity, exponential nonlinearity, and complex interface surface.

In this talk, current studies on SMPBE will be reviewed, including our recent web server (*smpbs.math.uwm.edu*) for solving a uniform SMPBE model and our special solution decomposition techniques. A new nonlinear successive over-relaxation algorithm will then be presented to solve the nonuniform SMPBE model based on the finite element approach. Next, our new SMPBE finite element software package will be introduced in this talk, together with numerical tests to well demonstrate the convergence and performance of our new nonlinear finite element iterative method for a well known Born ball test model and large proteins in several different ionic solvents with distinct ion sizes. This project was partially supported by the National Science Foundation, USA, through grant DMS-1226259.

Boundary Integral Equation Methods for Acoustic and Elastic Waves

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Abstract. In this talk, we discuss boundary integral equation methods for solving the two-dimensional fluid-solid scattering problem. Existence and uniqueness results for variational solutions of boundary integral equations equations are established. Since in all these boundary variational formulations, the hypersingular boundary integral operator associated with the time-harmonic Navier equation is a dominated integral operator, we also include a new regularization formulation for this hypersingular operator, which allows us to treat the hypersingular kernel by a wealkly singular kernel. A new computational approach is employed based on the series expansions of Hankel functions for the computation of weakly-singular boundary integral operators during the reduction of corresponding Galerkin equations into discrete linear systems. Numerical examples are presented to verify and validate the theoretical results.

Numerical Analysis of the Galerkin and Weak Galerkin Method for the Helmholtz Equation With High Wave Number

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Abstract. We study convergence property of the weak Galerkin method of fixed degree p and supercovergence property of the linear finite element method for the Helmholtz problem with large wave number.

1) Using a modified duality argument, we improve the existing error estimates of the WG method, in particular, the error estimates with explicit dependence on the wave number k are derived, which is shown that if $k(kh)^{p+1}$ is sufficiently small, then the pollution error in the energy norm is bounded by $O(k(kh)^{2p})$, which coincides with the phase error of the finite element method obtained by existent dispersion analyses.

2) For linear finite element method under certain mesh condition, we obtain the H^1-error estimate with explicit dependence on the wave number k and shown that the error between the finite element solution and the linear interpolation of the exact solution is superconvergent under the H^1-seminorm, although the pollution error still exists. We prove a similar result for the recovered gradient by polynomial preserving recovery (PPR) and found that the PPR can only improve the interpolation error and has no effect on the pollution error. Furthermore, we estimate the error between the finite element gradient and recovered gradient and discovered that the pollution error is canceled between these two quantities. Finally, we apply the Richardson extrapolation to the recovered gradient and demonstrate numerically that PPR combined with the Richardson extrapolation can reduce the interpolation and pollution errors simultaneously, and therefore, leads to an asymptotically exact *a posteriori* error estimator.

All theoretical findings are verified by numerical tests.

Contributed Talks

Modeling the Effect of Depression in Medical Illness

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Abstract. Aim of this article is to examine the impact of depression on the medical illness. The higher levels of depression influence disease severity risk via physiological mechanisms. Acute and chronic depression activates the hypothalamo-pituitary-adrenal (HPA) axis, which activates a flow of neuroendocrine alterations that have adverse effects on physical health and functioning. We introduce a mathematical model that incorporates the influence of depression on physical health. Our computational modeling suggests predictions about disease control. We study the stability of the equilibrium points. Numerical simulations of the model are presented to show the effect of various parameter values.

Wave Interaction With Structures in a Stratified Multilayer Fluid

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Abstract. We derive a linear system of equations governing the interaction of water waves with structures in a stratified multilayer fluid. Considering time-harmonic motions, we rewrite the problem as a spectral boundary-value problem, and give a suitable variational formulation. Trapped modes correspond to time harmonic oscillations of radian frequency ω and can be found as eigenfunctions of this problem. Our main result is a geometric condition relating the volume integrals of the submerged parts of the obstacles and the surface integrals along the parts of the surface/interface pierced by the obstacles and guaranteeing the existence of trapped modes. Several examples of structures (piercing the surface or the interfaces between layers) satisfying the condition and supporting trapped modes are given.

Preserving Positive Linear Discontinuous Finite Element Scheme for Spherical Neutron Transport Equations

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Abstract. The linear discontinuous finite element method for the spherical neutron transport equation will give negative angular flux. The preserving positive linear discontinuous finite element scheme has been constructed and this scheme can preserve 0 order moment and 1 order moment of neutron angular flux. The program for the spherical neutron transport problem has been established by the preserving positive linear discontinuous finite element scheme which uses the classical scheme by solving linear system of equations when the neutron angular flux is non-negative and adopts the preserving positive scheme by solving nonlinear system of equations when the neutron angular flux is negative. The numerical results show that the preserving positive scheme can give non-negative neutron angular flux which reflects the corresponding physical meaning and can give more accurate neutron angular flux to reducing numerical error.

Eddy Current Model for Nondestructive Evaluation with Thin Cracks

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Abstract. In this paper, we propose an approximate eddy current model for nondestructive evaluation. Interior cracks of large steel structures are very thin compared with the characteristic length of the system. Numerical methods usually necessitate very fine meshes to characterize the small thickness of cracks, and thus yield a very large number of degrees of freedom. The proposed model neglects the thickness of cracks and treats them as interior surfaces. The existence and uniqueness are established for the approximate solution upon introducing proper gauge conditions. The convergence of the approximate solution to the solution of the original eddy current problem is proved as the thickness of cracks tends to zero, and an error estimate is presented for homogeneous conducting materials. A coupled finite element method is proposed to solve the approximate problem. The well-posedness and the error estimate are proved for the discrete solution. Numerical experiments are carried out for engineering benchmark problems to validate the approximate eddy current model and to demonstrate the efficiency of the finite element method.

L^∞-error Estimates for the Finite Element Method of Parabolic Problems on Domains With Smooth Boundaries

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Abstract. We consider the finite element method for parabolic problems on a domain $\Omega \subset \mathbb{R}^N$ with smooth boundary $\partial\Omega \in C^\infty$ with the Neumann boundary condition. Since the boundary is smooth, we first approximate the domain by a polygonal domain Ω_h and then we triangulate it. In principle, we can perform the finite element method on the discrete domain Ω_h.

Our main goal is to establish an L^∞-error estimate for the solution approximated by the Lagrangian P^k-elements. Since $\Omega \triangle \Omega_h \neq \emptyset$ in general, we should pay close attention to the gap of the domains in analysis. In this talk, we show that the order of the L^∞-error is $O(h^2|\log h|)$ even if the degree k is bigger than one, although the order is $O(h^{k+1})$ when $\Omega = \Omega_h$. We also present the analyticity and maximal regularity for the discrete semigroup.

Nonconforming framework for GMsFEM (Generalized MultiScale Finite Element Method)

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Abstract. Multiscale problems prevail in many fields these days and have been creating important subjects in physics, industries, mathematics, and computational sciences. Simulation of multiscale problems keeping all the microscopic features is indomitable within the modern computational ca- pacity, and hence there should be some ways of reduction in the degrees of freedom which provide macroscale basis functions that represent microscale nature as much as possible. GMsFEM (Generalized Multiscale Finite Ele- ment Methods) is an efficient method to provide systematically computable reduced macroscale finite element basis functions each of which represents microscale nature of the medium. In this talk, a framework is introduced for nonconforming multiscale approach based on the GMsFEM. Snapshot spaces are constructed for each macroscale block. With suitable dimension reduction, offline spaces are constructed. Moment function spaces are then introduced to impose continuity among the local offline spaces, which results in nonconforming GMsFE spaces. Oversampling and randomized boundary condition strategies are considered. Steps for the nonconforming GMsFEM are given explicitly. Error estimates are derived. Numerical results are pre- sented to support the efficiency of the proposed approach.
This talk is based on the paper, “Nonconforming generalized multiscale fi- nite element methods”, Journal of Computational and Applied Mathematics (311), pp.215229, 2017, coauthored by Chak Shing Lee (Lawrence Livermore National Lab.)

Numerical Multi-symplectic Schemes for a Series of PDEs With Singularity

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Abstract. In recent years, PDEs with singular solutions have grasped the attention of numerous academics. Their interests in solving peakons and discontinuous problems have driven the development of many meaningful algorithms. Aiming at preserving some essential and inherent properties of these equations, we recur to the classical multi-symplectic theory. Also, this multi-symplectic formulation allows the application of various high-accuracy spatial discretization methods, such as the compact finite difference method, the Fourier pseudo-spectral scheme and the wavelet collocation method. Consequently, we are ready to present several combined multi-symplectic integrators for three intriguing PDEs (i.e., the two-component CamassaHolm equations, the HunterSaxton equation and the two-layer SerreGreenNaghdi equations) with singular solutions that preserve exactly the multi-symplecticity and known Hamiltonian structures under the correct choice of the boundary condition at the discrete level.

Error Analysis of Discontinuous Galerkin Finite Element Method for Optimal Control Problem Governed by the Transport Equation

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Abstract. This article discusses a priori and a posteriori error estimates of discontinuous Galerkin finite element method for optimal control problem governed by the transport equation. We use variational discretization concept to discretize the control variable and discontinuous piecewise linear finite elements to approximate the state and co-state variable. Based on the error estimates of discontinuous Galerkin finite element method for the transport equation, we get a priori and a posteriori error estimates for the transport equation optimal control problem. Finally, two numerical experiments are carried out to confirm the theoretical analysis.

Wet/Dry Tracking Well-balanced WAF Finite Volume Method for 2D Shallow Water Equations

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Abstract. In this work the well-balanced finite volume method with weighted average flux (WAF) is developed for 2D shallow water equations. The presented scheme can capture the wet/dry cells efficiently and also preserves non-negativity of water depths during time integration. The accuracy of the presented scheme is observed from various benchmark experiments, for both steady and unsteady flows. As illustrated numerically, the developed scheme is observed to have second order accuracy in space for smooth flow, when linear reconstruction is applied. In addition, to show the capability, the scheme is applied to simulate the real flooding occurred on the Chao Phraya river basin in Thailand, 2011, and the simulated results agree with the existing satellite data.

Finite Difference Approximation for Nonlinear Schrödinger Equations with Application to Blow-up computation

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Abstract. This talk presents a coherent analysis of the finite difference method to nonlinear Schrödinger (NLS) equations in one spatial dimension. We use the discrete H^1 framework to establish well-posedness and error estimates in the L^∞ norm. The nonlinearity $f(u)$ of a NLS equation is assumed to satisfy only a growth condition. We apply our results to computation of blow-up solutions for a NLS equation with the nonlinearity $f(u) = -|u|^{2p}$, p being a positive real number. Particularly, we offer the numerical blow-up time $T(h, \tau)$, where h and τ are discretization parameters of space and time variables. We prove that $T(h, \tau)$ converges to the blow-up time T_∞ of the solution of the original NLS equation. Several numerical examples are presented to confirm the validity of theoretical results. Furthermore, we infer from numerical investigation that the convergence of $T(h, \tau)$ is at a second order rate in τ if the Crank-Nicolson scheme is applied to time discretization.
This is a joint work with Prof. Norikazu Saito(Graduate School of Mathematical Sciences, The University of Tokyo).

A Five-Equation Model Based ALE Method for Compressible Multifluid Fluids

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Abstract. Multifluid flows can be found in a variety of science areas and practical applications, such as astrophysics and ICF (1-5). In this work, an ALE(Arbitrary Lagrangian-Eulerian) method coupled with five-equation model was proposed based on a novel closure model for multifluid mixed cells. The five-equation based ALE method is a two phase ALE method, including a Lagrangian phase and a rezone-remap phase (2, 3). In the first phase, a set of multifluid Lagrangian dynamics equations was discretized with a cell-centered Lagrangian scheme, and a multifluid Riemann solver was proposed by extending traditional Riemann solver for mixed cells with different materials. The rezone-remap technique was taken from (2, 3). Closure model for mixed cells is the key part of the five-equation based ALE method, and it was developed based on the analysis of variation relation of volume fraction in mixed cells with the assumption of isentropic compression. The proposed model is different from traditional five-equation model (4, 5), and it is consistent with the compressibility of different fluids in mixed cells. The five-equation based ALE method can be used to simulate multi-material flows with large deformation, which is an obstacle for many traditional ALE methods. Moreover, it can be used to simulate multiphase flows. A series of multi-material and multiphase test problems were simulated with the five-equation based ALE method, and numerical results agree well exact solutions and reference results.

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Exponential Fourier Collocation Methods for Solving First-order Differential Equations

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Abstract. In this talk, a novel class of exponential Fourier collocation methods (EFCMs) is presented for solving systems of first-order ordinary differential equations. These so-called exponential Fourier collocation methods are based on the variation-of-constants formula, incorporating a local Fourier expansion of the underlying problem with collocation methods. We discuss in detail the connections of EFCMs with trigonometric Fourier collocation methods (TFCMs), the well-known Hamiltonian Boundary Value Methods (HBVMs), Gauss methods and Radau IIA methods. We also analyse the properties of EFCMs.

Convergence of Numerical Solutions of Stochastic Integro-partial Differential Equations Driven by the Space-time White Noise

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Abstract. The work is concerned with the convergence of numerical methods for solving the stochastic integro-partial differential equation

$$\partial_t\psi - \Delta\partial_t^{1-\alpha}\psi = f + \varepsilon\dot{W}$$

for given $\alpha \in (0,1)\cup(1,2)$, where $\partial_t^{1-\alpha}\psi$ denotes the Caputo fractional derivative and $\dot{W}$ space-time white noise. For the above model, both the time-fractional derivative and the stochastic process result in low regularity of the solution. Hence, the numerical approximation of such problems and the corresponding numerical analysis are very challenging. In this work, the stochastic integro-partial differential equation is discretized by a backward Euler convolution quadrature in time and piecewise continuous linear finite element method in space for which the sharp-order convergence

$$\mathbb{E}\|\psi(\cdot,t_n) - \psi_n^{(h)}\|_{L^2(\mathcal{O})}^2 = \begin{cases} O\big(\tau^{1-\frac{\alpha d}{2}} + \ell_h h^{\frac{2}{\alpha}-d}\big) & \text{if } \alpha \in \Big[\dfrac{1}{2}, \dfrac{2}{d}\Big), \\ O\big(\tau^{1-\frac{\alpha d}{2}} + h^{4-d}\big) & \text{if } \alpha \in \Big(0, \dfrac{1}{2}\Big), \end{cases}$$

up to a logarithmic factor $\ell_h = \ln(2+1/h)$, is established for $\alpha \in (0, 2/d)$ in general d-dimensional spatial domains with nonsmooth initial data, where $\psi_n^{(h)}$ denotes the approximate solution at the nth time step, and $\mathbb{E}$ the expectation operator. Numerical results are presented to illustrate the theoretical analysis.

A Discontinuous Finite Element Method For Neutron Transport Equations on 3-D Unstructured Grids

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Abstract. Time-dependent neutron transport equation is a kind of important hyperbolic partial differential equation in nuclear science and engineering applications. High dimension neutron transport calculation include computing of space grid, angle direction, energy group and time step, is very complex and huge scale scientific calculation problem. Discontinuous finite element discrete ordinates (DFE-Sn) method is very efficient for solution of such equations especially while concerned with complicated physics including multimedia, larger grid distortion, complex initial and boundary conditions. In this paper, the discrete scheme of Sn discrete ordinate and discontinuous finite method 3-D unstructured tetrahedral meshes are presented. We developed a serial solver with DFE-Sn method to solve time-dependent neutron transport equations on unstructured tetrahedral grids. Domain decomposition scheme, parallel Sn sweep algorithm and multi-level Parallel Algorithm on unstructured grids are adopted to improve the efficiency, the parallel computation for the scheme is realized on MPI systems. Numerical experiments demonstrate the accuracy and efficiency of these methods.

Key words: discontinuous finite element method, transport equations, unstructured meshes, domain decomposition, sweep algorithm

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The Regularity Problem of Magneto-heating Coupling Model With Turbulent Convection Zone and the Flow Fields

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Abstract. In this report, Magneto-heating coupling model with turbulent convection zone and the flow fields is established. Our main work is to analyze the well-posedeness of this model with the regularity techniques. For magnetic field with turbulent convection zone and the flow fields, we consider the space $H(curl) \cap H(div)$ and for the heat equation, we consider the space $H_0^1(\Omega)$. The we present the weak formula of magneto-heating coupling model and establish the regularity problem. Using Roth's method, monotone theories of nonlinear operator, weak convergent theories, we prove that the limits of the solutions from Roth's method converge to the solutions of the regularity problem with proper initial data. With the help of the spacial regularity technique, we derive the results of the well-posedness of the original problems when the regular parameter $\epsilon \longrightarrow 0$.

Structure-preserving Methods for the Degasperis-procesi Equation

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Abstract. Our work gives several structure-preserving schemes for the Degasperis-Procesi (DP) equation. The DP equation has a bi-Hamiltonian structure and both Hamiltonian differential operators of the two structures are complex and non-local. For this sake, few structure-preserving schemes have been proposed so far. In our work, based on one structure of the bi-Hamiltonian structure, a symplectic scheme and two new energy-preserving schemes are constructed. We prove that two energy-preserving schemes obey the energy conservation law and the symplectic scheme obey the symplectic conservation law. Then some numerical experiments are implemented and the numerical results show that the three schemes do have the advantages in numerical stability, accuracy in long time computing and ability to preserve the invariants of the DP equation.

Numerical Methods for the Navier-stokes Equations With the Unilateral Boundary Condition of Signorini's Type

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Abstract. We consider the Navier-Stokes problem modelling the blood flow in aorta, where the velocity on the inflow boundary is given. The velocity and pressure on the outflow boundary cannot be prescribed, where the traction-free boundary condition has been applied in literature. However, the well-posedness is unknown for this model, and the unstable (or blow-up) numerical solutions are observed. To ensure the energy inequality, we propose a unilateral boundary condition of Signorini's type. The model problem is equivalent to an variational inequality, which satisfies the energy inequality. For numerical computation, we propose two methods: the penalty method and Lagrange multiplier method. We investigate the convergence of these two methods, and provide the implementation algorithms. Particularly, for the Lagrange multiplier method, we consider two implementation methods: the Uzawa method and Active/Inactive set method.

Stream 2

Mathematical Methods and Computation in Biology

Invited Talks

Mathematical Models of HIV-HPV Co-infection

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Abstract. HIV-infected patients are at increased risk of co-infection with human papilloma virus (HPV), and subsequent malignancies such as oral cancer. To determine the role of HIV-associated immune suppression on HPV persistence and pathogenesis, we developed models of HIV-HPV co-infection and used them to investigate the mechanisms underlying the modulation of HPV infection by HIV. We capture known immunological and molecular features, such as impaired HPV-specific effector T helper cell responses, and enhanced HPV infection in the presence of HIV. In this talk, I will show how the models can be used to make predictions regarding the HPV persistence in the oral mucosa. We predict that the permissive immune environment created by HIV, as well as molecular interactions between the two viruses, are the reasons for HPV persistence. We address the role of antiretroviral therapy and show that restoration of CD4 T cell count correlates with eventual HPV removal as seen in a recent clinical trial. Lastly, we investigate the effect of HPV on the size of the HIV latent reservoir.

Pattern Formation in a Nonlocal Model for Tumour Dynamics as a Result of the TGF-β Pathway

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Abstract. In this talk we present a nonlocal hyperbolic-parabolic model for the interactions between cancer cells, and their response to the versatile TGF-β molecules (which can have both pro-tumour and anti-tumour roles). With the help of this model, we investigate various hypotheses regarding the factors that might influence the evolution and structure of tumours in response to TGF-β molecules. We show that the model can explain the formation of aggregations of tumour cells (resembling tumour metastases) at positions in space further away from the main aggregation, and hypothesize that this is the result of the delicate balance between the tumour growth rate, the speed of tumour cells and the effects of TGF-β molecules on cell-cell adhesion. Finally, we perform a sensitivity analysis on some parameters associated with TGF-β dynamics, and use it to investigate the relation between tumour size and its metastatic spread.

Forced Waves of the Fisher-KPP Equation in a Shifting Environment

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Abstract. We consider the Fisher-KPP equation in a wavelike shifting environment. The questions of interest are whether such an environment can give rise to any forced wave solutions and how they (if exist) attract other solutions. This type of equation may arise from the consideration of pathogen spread in a classical susceptible-infected-susceptible epidemiological model of a host population where the disease impact on host mobility and mortality is negligible, and the mathematical question of interest corresponds to whether pathogen spread can keep pace with its host invasion. This talk is based on joint works with Yijun Lou and Jianhong Wu, and with Henri Berestycki.

The Complexity of Models of Waning Immunity in Infectious Diseases

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Abstract. Infectious diseases are complex. Mathematical models of complex systems can be very complex. A goal of mathematical modellers is to determine an 'optimal level' of biological complexity to incorporate into a model such that the model (1) can still be used to appropriately reflect the biological system being studied, and (2) will allow for mathematical and compulational analysis while minimizing uncertainty. In this talk I will discuss two mathematical models of infectious disease spread and waning immunity that are of particular interest to the infectious disease community. The first model is quite simple in formulation, but very rich in dynamics. The second model is very complex with many equations representing multiple levels of immunity status, but also very informative to the biological system. The drawbacks and benefits of both models will be discussed, including a discussion of the comuptational tools needed to study such systems. Implications to pertussis and influenza will be presented.

On the Periodic Logistic Map

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Abstract. In this paper, the famous logistic map is studied in a new point of view. We study the boundedness and the periodicity of the non-autonomous logistic map

$$x_{n+1} = r_n x_n (1 - x_n), \quad n = 0, 1, \cdots ,$$

where $\{r_n\}$ is a positive p-periodic sequence. The sufficient conditions are given to support the existence of asymptotically stable and unstable p-periodic orbits. This appears to be the first study of the map with variable parameter r.

Global Dynamics of an Infinite Dimensional Epidemic Model with Nonlocal State Structures

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Abstract. In this talk, I will describe a state-structured epidemic model for infectious diseases in which the state structure is nonlocal. The state is a measure of infectivity of infected individuals or the intensity of viral replications in infected cells. The model gives rise to a system of nonlinear integro-differential equations with a nonlocal term. I will show the well-posedness and dissipativity of the associated nonlinear semigroup by overcoming a lack of compactness due to the integral form of the equations. By establishing an equivalent principal spectral condition between the linearized operator and the next-generation operator, I will show that the basic reproduction number R_0 is a sharp threshold: if $R_0 < 1$, the disease-free equilibrium is globally asymptotically stable, and if $R_0 > 1$, the disease-free equilibrium is unstable and a unique endemic equilibrium is globally asymptotically stable. Our proof of the global stability of the endemic equilibrium utilizes a global Lyapunov function whose construction was motivated by the graph-theoretic method for coupled systems on discrete networks developed by Guo-Li-Shuai. This is a joint work with Drs. Zhipeng Qiu and Zhongwei Shen.

The Impact of Climate Warming and Spatial Heterogeneity on the Spreading of the West Nile Virus

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Abstract. This talk deals with mathematical models describing the dynamic of West Nile virus in North America. For the spatially-independent WNv model, the usual basic reproduction number R_0 is given and for the diffusive WNv model in a bounded domain, the basic reproduction numbers R_0^N, R_0^D are defined. To model and explore the expanding front of the infective region, a reaction-diffusion problem with free boundaries is proposed. The spatial-temporal risk index $R_0^F(t)$, which involves regional characteristic and time, is defined. Sufficient conditions for the virus to vanish or spread are given. Our results suggest that the spreading or vanishing of the virus depends on the initial number of infected individuals, the area of the infected region, the diffusion rate, and other factors. Some remarks on the basic reproduction numbers and the spreading speeds are presented and compared. Moreover, we establish a new WNv model to describe the impart of climate warming and spatial heterogeneity.

Evolution of Diffusion in a Mutation-selection Model

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Abstract. We consider an Integro-PDE model for a population structured by the spatial variables and a trait variable which is the diffusion rate. Competition for resource is local in spatial variables, but nonlocal in the trait variable. We show that in the limit of small mutation rate, the solution concentrates in the trait variable and forms a Dirac mass supported at the lowest diffusion rate. Hastings and Dockery et al. showed that for two competing species, the slower diffuser always prevails, if all other things are held equal. Our result suggests that their findings may well hold for a continuum of traits. This talk is based on joint work with Prof. King-Yeung Lam, Ohio State University.

Existence of Infinitely Many Radially Symmetric Solutions With Compact Support in $\mathbb{R}^N$ and Peaking Phenomenon

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Abstract. For a general elliptic problem $-\triangle u = g(u)$ in $\mathbb{R}^N$ which often arises in population dynamics, we show that all solutions have compact support. Furthermore, by using symmetric mountain pass theorem, we can prove that this problem has infinitely many radially symmetric solutions. In particular, a nonnegative least energy solution exists. With this result we study a singularly perturbed elliptic problem $-\epsilon^2\triangle u+|u|^{q-1}u = f(u)$ in a bounded domain Ω with $0 < q < 1$ and $u \in H_0^1(\Omega)$. By comparing energies, for any $y \in \Omega$, we find out that there exists a least energy solution u_ϵ, which concentrates around this point y as $\epsilon \to 0$. Conversely when ϵ is small, the boundary of the set $\{x \in \Omega \mid u_\epsilon(x) > 0\}$ is a free boundary, where u_ϵ is any nonnegative least energy solution.

Nonlocal PDE Models for Cell Interactions and Movement

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Abstract. Cells interact with each other in a non-local manner, through forming direct cell-cell contacts that can extend over multiple cell lengths. Classic and recent experiments have revealed how these interactions stimulate a variety of movement types, from adhesive binding that can drive the self-organisation of a population to contact inhibition of repulsion that leads to enhanced dispersal. In this talk I will describe continuous and discrete mathematical models for describing these non-local behaviours and demonstrate their capacity to show phenomena including pattern formation and travelling wave dynamics. I will discuss applications including neural crest dispersal and zebrafish pigmentation patterning and raise some mathematical challenges.

Population Dynamics of Epidemic and Endemic States of Drug-resistance Emergence in Infectious Diseases

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Abstract. The emergence and spread of drug-resistance during treatment of many infectious diseases continue to degrade our ability to control and mitigate infection outcomes using therapeutic measures. While the coverage and efficacy of treatment remain key factors in the population dynamics of resistance, the timing for the start of the treatment in infectious individuals can significantly influence such dynamics. We developed a between-host disease transmission model to investigate the short-term (epidemic) and long-term (endemic) states of infections caused by two competing pathogen subtypes, namely the wild-type and resistant-type, when the probability of developing resistance is a function of delay in start of the treatment. We characterize the behaviour of disease equilibria and obtain a condition to minimize the fraction of population infectious at the endemic state in terms of probability of developing resistance and its transmission fitness. For the short-term epidemic dynamics, we illustrate that depending on the likelihood of resistance development at the time of treatment initiation, the same epidemic size may be achieved with different delays in start of the treatment, which may correspond to significantly different treatment coverages. Our results demonstrate that early initiation of treatment may not necessarily be the optimal strategy for curtailing the incidence of resistance or the overall disease burden. The risk of developing drugresistance in-host remains an important factor in the management of resistance in the population. Joint work with Dina Knipl and Seyed Moghadas.

Free-virus and Cell-to-cell Transmission in Mathematical Models of Equine Infectious Anemia Virus Infection

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Abstract. Equine infectious anemia virus (EIAV) is a lentivirus in the retrovirus family that infects horses and ponies. Two strains, referred to as the sensitive strain and the resistant strain, have been isolated from an experimentally-infected pony. The sensitive strain is vulnerable to neutralization by antibodies whereas the resistant strain is neutralization-insensitive. The sensitive strain mutates to the resistant strain. EIAV may infect healthy target cells via free virus or alternatively, directly from an infected target cell through cell-to-cell transfer. The proportion of transmission from free-virus or from cell-to-cell transmission is unknown. A system of ordinary differential equations (ODEs) is formulated for the virus-cell dynamics of EIAV. In addition, a Markov chain model and a branching process approximation near the infection-free equilibrium (IFE) are formulated. The basic reproduction number is defined as the maximum of two reproduction numbers, one for the sensitive strain and one for the resistant strain. The IFE is shown to be globally asymptotically stable for the ODE model in a special case when the basic reproduction number is less than one. In addition, two endemic equilibria exist, a coexistence equilibrium and a resistant strain equilibrium. It is shown that if the basic reproduction number > 1, the infection persists with at least one of the two strains. However, for small infectious doses, the sensitive strain and the resistant strain may not persist in the Markov chain model. Parameter values applicable to EIAV are used to illustrate the dynamics of the ODE and the Markov chain models. The examples highlight the importance of the proportion of cell-to-cell versus free-virus transmission that either leads to infection clearance or to infection persistence with either coexistence of both strains or to dominance by the resistant strain.

Sensitivity of the General Rosenzweig–MacArthur Predator-Prey Model to the Mathematical Form of the Functional Response: a Bifurcation Theory Approach

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Abstract. The equations in the Rosenzweig–MacArthur predator-prey model have been shown to be sensitive to the mathematical form used to model the predator response function even if the forms used have the same basic shape: zero at zero, monotone increasing, concave down, and saturating. Here, we revisit this model to help explain this sensitivity in the case of Holling type II, Ivlev, and Trigonometric response functions. We consider both the local and global dynamics and determine the possible bifurcations with respect to variation of the carrying capacity of the prey, a measure of the enrichment of the environment. We give an analytic expression that determines the criticality of the Andronov-Hopf bifurcation, and prove that although all three forms can give rise to supercritical Andronov-Hopf bifurcations, only the Trigonometric form can also give rise to subcritical Andronov-Hopf bifurcation and has a saddle node bifurcation of periodic orbits giving rise to two coexisitng limit cycles, providing a counterexample to a conjecture of Kooij and Zegeling (1996) and a related result in a paper by Attili and Mallak (2006). We also revisit the ranking of the functional responses, according to their potential to destabilize the dynamics of the model and show that given data, not only the choice of the functional form, but the choice of the number or position of the data points can influence the dynamics predicted. Similar considerations in an analogous predator-prey model in the chemostat will also be discussed.

A Mathematical Model of Ebola Virus Disease: Using Sensitivity Analysis to Determine Effective Intervention Targets

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Abstract. In December of 2013, an outbreak of Ebola began in the West African country of Guinea and later spread to Sierra Leone and Liberia. Health Organisations like the US Centers for Disease Control and the World Health Organization were tasked with providing aid to end the outbreak. We create an SEIR compartmental model of Ebola with a fifth compartment for the infectious deceased to model the dynamics of an Ebola outbreak in a village of a thousand people. We analyse the disease-free equilibrium of the model and formulate an equation for the eradication threshold R_0. Sensitivity analyses points us in the direction of the transmission probability and the contact rate with infectious individuals as targets for intervention. We model the effect that vaccination and quarantine, together and separately, have on the outcome of the Ebola epidemic. We find that quarantine is a very effective intervention, but when combined with vaccination it can theoretically lead to eradication of the disease. The model is extended to include a theoretical disease-modifying vaccine and conditions established that prevent the vaccine from making the epidemic worse.

Dengue Control: Data and Modeling Analyses

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Abstract. In this talk, our recently works concerning the dengue disease control will be introduced and discussed. In particular, based on the data sets what we have done on this topic and what we aim to go further will be introduced in more detail. Moreover, we would also like to introduce the characteristics of data sets, main methods and results obtained in those works, and the effectiveness of integrated control measures related to the dengue diseases will be also discussed based on the data and modeling analyses.

Mathematical Modeling of the Spread of Wolbachia for Dengue Control

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Abstract. In the fight against world-wide plague of dengue and of other arboviruses, innovative vector-control techniques have been developed. Among them, the use of the bacteria Wolbachia has received considerable attention. In fact, this maternally transmitted bacteria blocks the transmission of some arboviruses from mosquitoes to human. Therefore, releases of Wolbachia-infected mosquitoes are performed in the field. An objective of these releases is the replacement of the mosquitoes population by a Wolbachia-infected population, unable to transmit diseases. In this talk, we will consider the modeling and a mathematical study of the spread of this bacteria into a host population. From a mathematical point of view, this study boils down to look for traveling waves for some reaction-diffusion model and the conditions to ignite the propagation. Moreover, we will also consider the influence of heterogeneity in the environment in the invasion process.

A PDE System Modeling the Growth of Phytoplankton Consuming Inorganic Carbon With Internal Storage

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Abstract. This talk presents a PDE system modeling the growth of a single species population consuming inorganic carbon that is stored internally in a poorly mixed habitat. Inorganic carbon takes the forms of "CO2" (dissolved CO2 and carbonic acid) and "CARB" (bicarbonate and carbonate ions), which are substitutable in their effects on algal growth. We first establish a threshold type result on the extinction/persistence of the species in terms of the sign of a principal eigenvalue associated with a nonlinear eigenvalue problem. If the habitat is the unstirred chemostat, we add biologically relevant assumptions on the uptake functions and prove the uniqueness and global attractivity of the positive steady state when the species persists.
This talk is based on my recent work joint with Drs. Sze-Bi Hsu and King-Yeung Lam.

Comprehensive Effect of ART on HIV Infection: Insight From Multi-scale Models

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Abstract. The antiretroviral therapy (ART) has been shown to be effective in slowing down the progression to AIDS. Estimating the impact of ART is important to guide treatment regime both at the individual level and at the population level. Within-host viral dynamic model and between-host transmission dynamic model are employed to investigate comprehensive effect of ART on HIV infection. In this talk, I shall initially extend within-host viral dynamic model to include general time-dependent incidence to faithfully mimic the whole natural HIV disease progression and predict life expectancy. A summative treatment benefit index is defined to examine the comprehensive effect of ART on life expectancy and to predict the time of virological failure, which may provide information about the time for switching to the next-line regimen without resistance testing. Then, to investigate the impact of various treatment regimes on HIV infection at the population level, an individual based simulation model was proposed to couple between-host transmission dynamics with within-host viral dynamics. In particular, we will focus on the effects of different timings to start ART or drug efficacy on HIV new infection among men who have sex with men (MSM). Some key control strategies will then be proposed.
Joint work with Xiaodan Sun, Sanyi Tang, Yicang Zhou, Ning Wang, Zhihang Peng, Jianhong Wu

A Malaria Transmission Model with Temperature-Dependent Incubation Period

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Abstract. Malaria is an infectious disease caused by Plasmodium parasites and is transmitted among humans by female Anopheles mosquitoes. Climate factors have significant impact on both mosquito life cycle and parasite development. To consider the temperature sensitivity of the extrinsic incubation period (EIP) of malaria parasites, we formulate a delay differential equations model with a periodic time delay. We derive the basic reproduction ratio R0 and establish a threshold type result on the global dynamics in terms of R0. More precisely, we show that the unique disease-free periodic solution is globally asymptotically stable if R0<1, and the model system admits a unique positive periodic solution which is globally asymptotically stable if R0>1. Numerically, we parameterize the model with data from Maputo Province, Mozambique and simulate the long term behavior of solutions. The simulation result is consistent with the obtained analytic result. In addition, we find that using the time-averaged EIP may underestimate the basic reproduction ratio.
This talk is based a joint work with Xiunan Wang.

On a Diffusive Host-pathogen System With Different Dispersal Rates

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Abstract. In this talk, I will report some recent results (joint work with Dr Yixiang Wu) on a diffusive host-pathogen model with heterogeneous parameters and distinct dispersal rates for the susceptible and infected hosts. In addition to global existence of solution, existence of a global attractor, we also discuss the threshold dynamics in terms of the basic reproduction number R0 which is identified as the spectral radius of a linear operator in the appropriate functions space. We show that if R0<1, the pathogen free equilibrium is globally stable, and if R0>1, the solution of the model is uniformly persistent and there exists a positive steady state. In the latter case, we also explore the asymptotic profiles of the endemic steady state as the dispersal rate of the susceptible or infected hosts approaches zero. Our result suggests that, when the diffusion rate tends to zero, the infected hosts concentrate at certain points which can be characterized as highest-risk locations.

Contributed Talks

Planar Morphometrics via Teichmüller Mappings

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Abstract. In this talk, we present a new method for planar morphometrics using Teichmüller mappings. Teichmüller mapping produces a natural 1-1 correspondence between two planar biological shapes with prescribed landmarks and shape boundaries exactly matched. Hence, our method can be used for accurately comparing planar shapes. Furthermore, a measure of similarity can be derived from Teichmüller mappings. By analyzing the similarity matrix formed via pairwise Teichmüller mappings between a set of shapes, a shape classification scheme is developed. Experiments on Drosophila wings suggest that our method effectively captures the phenotypic features of shapes.

Fast High Order Methods for Electromagnetic Scattering by Large Cavities

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Abstract. The analysis of the electromagnetic scattering by cavities have attracted tremendous interest by the engineering and mathematical communities for their significant industrial and military applications. When the wave number is large, or the cavity becomes large compared to the wavelength of the fields, the problem will be challenging on account of the highly oscillatory nature of the fields.

In this talk, we first give a fast high order method for the electromagnetic scattering by a single open cavity filled with the homogeneous medium in terms of the discrete Fourier transform and Gaussian elimination. On this basis, we apply the immersed interface method to develop a high order algorithm for the cavity filled with the inhomogeneous media. Moreover, for the multiple cavity scattering, we consider the coupling of transparent boundary conditions on the apertures and construct a fast high order method for the multiple cavity scattering problem.

Stream 3
Computational Optimization

Invited Talks

A General Double-proximal Gradient Algorithm for d.c. Programming

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Abstract. The possibilities of exploiting the special structure of d.c. programs, which consist of optimizing the difference of convex functions, are currently more or less limited to variants of the DCA proposed by Pham Dinh Tao and Le Thi Hoai An in 1997. These assume that either the convex or the concave part, or both, are evaluated by one of their subgradients.

In this talk we propose an algorithm which allows the evaluation of both the concave and the convex part by their proximal points. Additionally, we allow a smooth part, which is evaluated via its gradient. In the spirit of primal-dual splitting algorithms, the concave part might be the composition of a concave function with a linear operator, which are, however, evaluated separately.

For this algorithm we show that every cluster point is a solution of the optimization problem. Furthermore, we show the connection to the Toland dual problem and prove a descent property for the objective function values of a primal-dual formulation of the problem. Convergence of the iterates is shown if this objective function satisfies the Kurdyka - Łojasiewicz property. In the last part, we apply the algorithm to an image processing model.

The talk relies on a joint work with Sebastian Banert.

Using Spherical t-designs and ℓ_q minimization for Recovery of Sparse Signals on the Sphere

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Abstract. This paper considers the recovery of sparse signals on the sphere from their low order, potentially noisy, Fourier-Laplace coefficients. The approach uses spherical t-designs (sets of points on the sphere for which equal weight cubature is exact for all spherical polynomials of degree at most t) to formulate a constrained $\ell_q(0 < q \leq 1)$ minimization model. We show that well-conditioned spherical t-designs and ℓ_q minimization have effective properties for recovery of sparse signals on the sphere. A key role is played by the separation of the underlying point set on which the discrete signal resides. Numerical examples and a discussion of super-resolution are provided.

Online Optimization, Smoothing, and Worst-case Competitive Ratio

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Abstract. In Online Optimization, the data in an optimization problem is revealed over time, and at each step a decision variable needs to be set without knowing the future data. This setup covers online resource allocation, from classical inventory problems to the ‘Adwords’ problem popular in online advertising.

We discuss several algorithms for a class of online convex optimization problems, focusing on the algorithms’ competitive ratio, i.e., the ratio of the objective achieved by the online algorithm to that of the optimal offline sequence of decisions. We discuss a bound on this ratio for a primal-dual greedy algorithm, for the worst-case input sequence. We show how smoothing the objective can improve this bound, and for separable functions, how to seek the optimal smoothing by solving a convex optimization problem. This approach allows us to design effective smoothing, customized for a given cost function.

The LP-Newton Method and Recent Developments

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Abstract. The LP-Newton method was designed for systems of nonlinear equations having nonisolated degenerate solutions. In particular, such problems may arise from complementarity systems. The name of the method stems from the fact that its subproblems are linear programs. Under quite weak conditions, the LP-Newton method converges locally with quadratic rate. Besides some basic results we will highlight the role of error bounds and present applications to piecewise smooth systems and generalized Nash equilibrium problems. Moreover, we will detail ideas for the globalization of the LP-Newton method.

Generalized Nash Equilibrium Problems In Banach Spaces: Theory, Nikaidoisoda-based Path-following Methods, And Applications

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Abstract. A class of noncooperative Nash equilibrium problems is presented, in which the feasible set of each player is perturbed by the decisions of their competitors via a convex constraint. In addition, for every vector of decisions, a common state variable is given by the solution of an affine linear equation. The resulting problem is therefore a generalized Nash equilibrium problem (GNEP). The existence of an equilibrium for this problem is demonstrated, and first-order optimality conditions are derived under a constraint qualification. An approximation scheme is proposed, which involves the solution of a parameter-dependent sequence of standard Nash equilibrium problems. An associated path-following strategy based on the NikaidoIsoda function is then proposed. Function-space-based numerics for parabolic GNEPs and a spot-market model are developed.

New Results for Anderson Acceleration: Theory and Applications

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Abstract. Anderson Acceleration was designed to accelerate Picard iteration in the context of electronic structure computations in Physics. In this talk we will review our results on convergence, in particular our recent work on the cases in which he fixed point map is corrupted with errors. We consider uniformly bounded errors and stochastic errors with infinite tails. We prove local improvement results which describe the performance of the iteration up to the point where the accuracy of the function evaluation causes the iteration to stagnate. We will also discuss the EDIIS variation of the method, which is used in quantum chemistry applications.

Mixed-Integer PDE Constrained Optimization

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Abstract. We introduce a new class of complex optimization problems that can be formulated as optimization problems constrained by partial differential equations (PDEs) with integer decision variables. Examples include the remediation of contaminated sites and the maximization of oil recovery; the design of next generation solar cells; the layout design of wind-farms; the design and control of gas networks; disaster recovery; and topology optimization. We will present emerging applications of mixed-integer PDE-constrained optimization, review existing approaches to solve these problems, and highlight their computational and mathematical challenges. We introduce a new set of benchmarks for this challenging class of problems, and present some early numerical experience using both mixed-integer nonlinear solvers and simple rounding heuristics.

Algorithmic Development for Computing B-stationary Points of a Class of Nonsmooth DC Programs

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Abstract. In the first part of this talk, we study convex-constrained nonsmooth DC program in which the concave summand of the objective is an infimum of possibly infinitely many smooth concave functions. We propose some algorithms by using the extrapolation technique for this problem, and analyze their global convergence, sequence convergence and also convergence rate.

In the second part we consider a class of DC constrained nonsmooth DC program. We propose quadratic penalty, exact penalty and augmented Lagrangian methods for solving this problem. Also, we show that the methods either finds an approximate B-stationary point or converges to a B-stationary point.

Dynamic Resource Allocation for Energy Efficient Transmission in Digital Subscriber Lines

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Abstract. Linear matrix precoding, also known as vectoring, is a well-known technique to mitigate multiuser interference in the downlink Digital Subscriber Line (DSL) transmission. While effective in canceling interference, vectoring does incur major computational overhead in terms of a matrix vector multiplication at each data frame, resulting in significant energy consumption when the number of lines is large. To facilitate energy efficient transmission, it has been recently proposed (in the G.fast standard) that each data frame is divided into a normal operating interval (NOI) and a discontinuous interval (DOI). In the NOI, all lines (or users) are involved in a common vectoring group, which requires a large matrix precoder, while in a DOI, the lines are subdivided into multiple small non-overlapping vectoring subgroups, which are transmitted in a TDMA manner within the data frame. Because of the use of small matrix precoders for the small vectoring subgroups in DOI, the energy efficiency can be significantly improved. In this paper, we consider several key dynamic resource allocation (DRA) problems in DSL: given the instantaneous buffer state, determine the number of transmission opportunities allocated to each line, the optimal NOI and DOI size in each data frame as well as the optimal grouping in DOI. We formulate these optimal DRA problems and propose efficient real-time algorithms for three main tasks: given a data frame, allocate transmission opportunities for all lines, design grouping strategy in DOI, and optimally adjust the durations of the NOI and the vectoring subgroups in the DOI. The simulation results show the efficiency and the effectiveness of our algorithms.

This is joint work with Nan Zhang, Zhiqiang Yao, Yixian Liu and Stephen Boyd

Tame Big Convex Optimization Problems Using the Second Order Sparsity

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Abstract. Big convex optimization models are ubiquitous in machine learning, statistics, finance, signal processing, imaging science, geophysics and many other areas. Concerned with the huge computational burdens of the interior point methods (IPMs) for solving big scale problems, many researchers and practitioners tend to believe that the first order methods such as the accelerated proximal gradient methods and the alternating direction methods of multipliers are the only option for the rescue. While these first order methods have enjoyed very successful stories for some interesting applications, they also encounter enormous numerical difficulties in dealing with many real data problems of big scales even only with a low or moderate solution quality. New ideas for solving these problems are highly sought both in practice and academic research. In this talk, we shall demonstrate how the second order sparsity property exhibited in big sparse optimization models can be intelligently explored to overcome the mentioned difficulties either in IPMs or in the first order methods. One critical discovery is that the second order sparsity allows one to solve sub-problems at costs even lower than many first order methods. For the purpose of illustration, we shall present highly efficient and robust semismooth Newton based augmented Lagrangian methods for solving various lasso and support vector machine models.

First Order Methods Beyond Lipschitz Gradient Continuity

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Abstract. A central assumption in first order methods (FOM) is to require the differentiable part of an objective function to have a Lipschitz continuous gradient, and hence precludes the use of FOM in many important applications. We introduce a new and simple framework which allows to circumvent the intricate question of Lipschitz continuity of the gradient. It naturally translates into a new descent Lemma, and a corresponding first order scheme for the generic convex composite minimization model. We prove for the resulting scheme a global sublinear rate of convergence, and as a by-product, global pointwise convergence is also derived. This provides a new path for tackling a broad spectrum of problems arising in key applications which were until now, considered as out of reach via first order methods. We illustrate this potential by showing how our new framework can be applied to derive new and simple schemes for a broad class of problems. This talk is based on joint work with H.Bauschke and J. Bolte.

SDPNAL+: A Matlab Software Package for Large Scale SDPs With a User-friendly Interface

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Abstract. SDPNAL+ is a Matlab software package that implements a 2-phase augmented Lagrangian based method to solve large scale semidefinite programming problems with bound constraints. The implementation was initially based on a majorized semi-smooth Newton-CG augmented Lagrangian method, but we subsequently put it within an inexact symmetric Gauss-Seidel based semi-proximal ADMM/ALM framework for the convenience of deriving simpler stopping conditions. The basic code is written in Matlab, but some computational intensive subroutines written in C language are incorporated via Mex files. We also design a basic interface for users to input their SDP models into the solver. Numerous problems arising from combinatorial optimization and binary integer quadratic programming problems have been tested to evaluate the performance of the solver. Extensive numerical experiments show that the proposed method is quite efficient and robust. In particular, we are able to solve a very large SDP problem with over 12 million constraints and a positive semidefinite matrix variable of dimension over 9200 in about 10 hours on a modest desktop PC.

PDE-constrained Optimization Under Uncertainty Using Low-rank Tensor Methods

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Abstract. We develop low-rank tensor methods for solving inequality-constrained optimization problems with PDEs involving uncertain parameters, focussing on control-constrained optimal control of PDEs under uncertainty. The proposed optimization methods perform all computations within a low-rank tensor format (Hierarchical Tucker or Tensor Train) and by this can overcome the curse of dimensionality. We discuss theoretical and numerical aspects, including techniques for adaptive inexactness control in a trust-region framework. We also plan to address possible extensions towards coherent risk measures (especially CVaR) and state constraints.

Robust Nonconvex PDE-Constrained Optimization based on Second Order Approximation Techniques and Reduced Order Models

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Abstract. We consider robust optimization techniques for nonconvex PDE-constrained problems involving uncertain parameters. The parameters are assumed to be contained in a given uncertainty set. This type of robust optimization problems are difficult to treat computationally and hence suitable approximations and solution methods are required. We propose and investigate an approximate robust formulation that employs a quadratic approximation (or only a linear approximation when appropriate) and can be solved efficiently by using a full-space formulation as mathematical program with equilibrium constraints (MPEC) or a reduced formulation. Moreover, we consider the application of reduced order models with a posteriori error estimation within the optimization method to reduce the number of required PDE-solves during the optimization. We show applications to the robust geometry optimization of a permanent magnet synchronous motor and to the robust geometry optimization of load-carrying structures governed by the elastodynamic equations.
This is joint work with Oliver Lass and Philip Kolvenbach, TU Darmstadt.

Epiconvergence, Moreau Envelope, and Generalized Linear-quadratic Functions

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Abstract. This work deals with generalized linear-quadratic functions and their Moreau envelopes. In finite dimensions, on a metric space defined by Moreau envelopes, we consider the epigraphical limit of a sequence of quadratic functions and categorize the results. We explore the question of when a quadratic function is a Moreau envelope of a generalized linear-quadratic function; characterizations involving nonexpansiveness and Lipschitz continuity are established. Some work by Hiriart-Urruty and by Rockafellar & Wets are generalized.
Joint work with C. Planiden.

Low-rank Matrix Completion (LRMC) Using Nuclear Norm (NN) with Facial Reduction (FR)

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Abstract. Minimization of the NN is often used as a surrogate, convex relaxation, for solving LRMC problems. The minimum NN problem can be solved as a trace minimization semidefinite program (SDP). The SDP and its dual are regular in the sense that they both satisfy strict feasibility. FR has been successful in regularizing many problems where strict feasibility fails, e.g., SDP relaxations of discrete optimization problems such as QAP, GP, as well as sensor network localization. Here we take advantage of the structure at optimality for the NN minimization and show that even though strict feasibility holds, the FR framework can be successfully applied to obtain a proper face that contains the optimal set. This can dramatically reduce the size of the final NN problem while guaranteeing a low-rank solution. We include numerical tests for both exact and noisy cases.

On Solving Non-Lipschitz Nonconvex Programs: Necessary Optimality Conditions, Exact Penalization and an Augmented Lagrangian Method

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Abstract. Including a non-Lipschitz term in the objective function has significantly enlarged the applicability of standard nonlinear programs. In particular it has recently been discovered that when the objective function belongs to a certain class of non-Lipschitz functions, local minimizers are often sparse. However when the objective function is not Lipschitz, standard constraint qualifications are no longer sufficient for Kaush-Kuch-Tucker (KKT) conditions to hold at a local minimizer, let alone ensuring an exact penalization. In this paper we extend quasi-normality and relaxed constant positive linear dependence condition to allow the non-Lipschitzness of the objective function and show that they are sufficient for KKT condition to be necessary for optimality. Moreover, we derive exact penalization results for the following two special cases: when the non-Lipschitz term in the objective function is the sum of a composite function of a separable lower semi-continuous function with a continuous function and an indicator function of a closed subset and when the non-Lipschitz term is the sum of a continuous function and an indicator function of a closed subset. An augmented Lagrangian method is also proposed to find the stationary points of the problem.

How Randomization Help to Solve Optimization Problems

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Abstract. Optimization algorithms have been recently applied to solve problems where data possess certain randomness, partly because data themselves contain randomness in a big-data environment or data are randomly sampled from their populations. It has been shown that data randomness typically makes algorithms run faster in the so-called "average behavior analysis". In this short talk, we give an example to show that a general non-convex quadratically constrained quadratic optimization problem, when data are randomly generated and the variable dimension is relatively higher than the number of constraints, can be globally solved with high probability via convex optimization algorithms. We give another example so show that a deterministic algorithm may not converge in general. But if we introduce a randomization scheme into the algorithm, then it converges in expectation.

A Partial Affine-scaling Method for Linearly Constrained Optimization Problems

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Abstract. We present a partial first-order affine-scaling method for solving smooth optimization with linear inequality constraints. At each iteration, the algorithm considers a subset of the constraints in order to reduce computation complexity. We prove the global convergence of the method and also studies convergence rate for special cases. Numerical results are reported to show the efficiency of our algorithm.
Joint work with Ran GU

Constrained Non-Convex Block Optimization over Manifolds

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Abstract. In this talk we shall present some new results on non-convex optimization models over manifolds with linear constraints. We introduce some ADMM (Alternating Direction Method of Multipliers) style algorithms for a block optimization model where the objective is non-convex and each block variables are elements of some given manifolds. Moreover, there are also linear constraints linking all the variables. Such models arise naturally in tensor optimization with constraints, including approximative Tucker decomposition with constraints. Iteration complexity bounds for the iterates converging to a stationary solution are presented, together with preliminary numerical results.

Contributed Talks

Parametric Optimization of Large-scale Industrial and Scientific Problems in Distributed and Parallel Environments

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Abstract. Geneva (Grid-enabled evolutionary algorithms) is a library of parametric optimization algorithms covering execution on parallel devices ranging from GPGPUs and many-core systems over clusters to Grids and Clouds. Five optimization algorithms, including particle swarms and evolutionary algorithms have been implemented, and best solutions from one algorithm may be transferred to another. Design-of-experiments may be conducted. Parallelization is mostly transparent to the user, leaving little work for him to be done for execution on a wide range of devices, once the optimization problems has been defined. The entire library is available as Open Source and targets particularly problems with very long running, computationally expensive optimization problems, often involving simulations. The definition of optimization problems may involve constraints between multiple parameters. Geneva was originally developed for and used in science, particularly particle physics (hence the name) and solid state physics, but is today also used in the automotive industry. It is being marketed as commercially supported Open Source software by a spinoff from Karlsruhe Institute of Technology. The presentation covers the architecture of Geneva and introduces use-cases.

On the Efficient Computation of The Projection Onto the Doubly Nonnegative Cone

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Abstract. Projecting a given matrix onto the the doubly nonnegative cone, which is the intersection of the positive semidefinite cone and the componentwise nonnegative cone, is a challenging task numerically since the constraints may likely to be degenerate. In this talk, we demonstrate an efficient two-stage algorithm for solving this projection problem to a very high accuracy. An accelerated proximal gradient method is adopted in the first stage with the aim of generating a reasonably good initial point, and an augmented Lagrangian is designed in the second stage with the subproblems being solved by the semismooth Newton-CG method. Extensive numerical results on various large scale instances show that our algorithm is efficient and robust in obtaining accurate solutions.

Unified SVRG for Optimization on Riemannian Manifold

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Abstract. In this paper, we propose a unified stochastic variance reduced gradient (SVRG) method for empirical risk minimization over Riemannian manifold. Existing SVRG methods on manifold usually consider a specific retraction operation, and involve additional computational costs such as parallel transport or vector transport. The unified SVRG (U-SVRG) we propose in this paper handles general retraction operations, and do not need additional computational costs mentioned above. We analyze the iteration complexity of U-SVRG for obtaining an ϵ-stationary point and its local linear convergence by assuming the Łojasiewicz inequality, which naturally holds for PCA and holds with high probability for matrix completion problem. We also incorporate the Barzilai-Borwein step size and design a very practical U-SVRG-BB method. Numerical results on PCA and matrix completion problems are reported to demonstrate the efficiency of our methods.

Conjugate Gradient Approach for Linear Optimal Control Problem with Model-Reality Differences

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Abstract. In this paper, the conjugate gradient approach, which is an optimization technique, is proposed to solve the linear optimal control problem. In our approach, a different linear optimal control model, which is adding the adjusted parameters into the model used, is to be solved. At the beginning, an expanded optimal control problem is introduced, where the Hamiltonian function is defined, and then, a set of the corresponding necessary conditions is derived. As a result, a modified model-based optimal control problem is defined. Follow from this, a formulation of the optimization problem is made. During the calculation procedure, the conjugate gradient algorithm is employed to solve the optimization problem, in turn, updating the adjusted parameters repeatedly to obtain the optimal solution of the model used. Once the convergence is achieved, the iterative solution approximates to the correct optimal solution of the original optimal control problem despite model-reality differences. For illustration, two examples are demonstrated to show the result. In conclusion, the efficiency of the approach proposed is highly presented.

Keywords: Conjugate Gradient, Linear Optimal Control, Iterative Solution, Adjusted Parameters, Model-Reality Differences

Application of Robust Optimization Approach for Supply Chain Design, Capacity Planning and Optimization under Risk and Uncertainty

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Abstract. The global competition, short life span of products, unstable economy situations are some of the few reasons which kept the modern supply chains vigorous as well as vulnerable to various risk and uncertainties. The presence of such factors is inevitable and proactive approaches are method to handle them efficiently. In this study, supply chain network design phase is identified to incorporate the robustness while designing the network. A mathematical model is derived to capture the supply side risks as well as demand uncertainty for a multi-echelon supply chain configuration. The formulation comes out to be a mixed integer linear programming (MILP). To address the uncertainty and its impact on the network, robust optimization (RO) approach is incorporated and used. The managerial insights for the supply chain design, capacity planning and optimization under risk and uncertainty is generated by carrying out numerous numerical experiments. The result of the study shows that supply chain designed and capacity planning based on RO perform ahead of the curve in worst cases of risk and uncertainty realizations.

Keywords: Supply Chain Design, Capacity Planning, Supply Chain Risk, MILP, Robust Optimization

On a Special Robust Optimization Problem-design of Secure Beamforming in Wiretap Networks With Channel Uncertainty

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Abstract. We consider a robust optimization problem arising from wireless communications. In a relay-aided wiretap network, we minimize the total relay transmit power, while requiring that the achieved rate at the supported users are above some thresholds, and that at the eavesdropper is below a standard. This problem is modeled as an optimization problem with one robust constraint. We relax the robust constraint to the worst case constraint, and solve the variables and the robust parameters alternatively, where the subproblems are solved optimally and the convergence of the objective function is proved. Simulation results show that our algorithm outperforms the state of the art, and has little loss compared to the result with perfect channel state information.

Stream 4
Tensor Computation

Invited Talks

Tensorization as a Concept

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Abstract. Various applications include the use of tensor techniques on multiway data. Multiway or tensor data, such as video data or spatial data, often originate naturally from measurements. One can also include additional variables in multiway experiment design, such as measuring multichannel biomedical signals across different subjects or including different emotions or light intensities in face databases.

Multilinear algebra has been used before without the availability of (low-rank) tensor data. For example, in blind signal separation and clustering, only a single matrix of observations is given. In other applications, a multivariate function is given but evaluation in a grid is not possible or meaningful. Typically, and possibly without knowing, a transformation to tensor data takes place, which we define as tensorization.

A straightforward form of tensorization is reshaping or folding, also known as segmentation or quantization. This is only one of many tensorization techniques. Others include the outer product, Hankelization and higher-order statistics such as cumulants and moments. Also higher-order derivatives of multivariate functions have tensorization abilities.

We present the concept of tensorization and give an overview of various tensorization techniques in a structured and insightful way, many of which have been presented disparately in the literature. We explain why these tensorization-based techniques work and when tensor decompositions can be applied in a meaningful manner. Furthermore, we provide links between multivariate functions, polynomials and tensors.

Spectral and Nuclear Norms of Higher-Order Tensors

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Abstract. We discuss several mathematical and computational properties of the spectral and nuclear norms for higher-order tensors. We show that like tensor rank, tensor spectral and nuclear norms are dependent on the choice of base field — the value of the spectral and nuclear norm of a real 3-tensor depends on whether we regard it as a real 3-tensor or a complex 3-tensor with real entries. We show that every tensor has a nuclear norm attaining decomposition and every symmetric tensor has a symmetric nuclear norm attaining decomposition. There is a corresponding notion of nuclear rank that, unlike tensor rank, is upper semicontinuous. We establish an analogue of Banach's theorem for tensor spectral norm and Comon's conjecture for tensor rank — for a symmetric tensor, its symmetric nuclear norm always equals its nuclear norm. We show that computing tensor nuclear norm is NP-hard in several sense. Deciding weak membership in the nuclear norm unit ball of 3-tensors is NP-hard, as is finding an ε-approximation of nuclear norm for 3-tensors. In addition, the problem of computing spectral or nuclear norm of a 4-tensor is NP-hard, even if we restrict the 4-tensor to be bi-Hermitian, bisymmetric, positive semidefinite, nonnegative valued, or all of the above. We discuss some simple polynomial-time approximation bounds and applications to quantum information theory.

Tensor Representation Of Spin States

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Abstract. The rise of quantum information theory has led to an increased interest in simple representations of general mixed states of systems with finite dimensional Hilbert spaces. Known for a long time is the case of two–level systems, such as spins-1/2. For such systems the density matrix can be expressed in a basis formed of the three Pauli matrices together with the identity matrix, leading to a parametrization in terms of a vector in $\mathbb{R}^3$. Pure states correspond to points on the unit sphere in $\mathbb{R}^3$, the so-called Bloch sphere, and mixed states fill the inside of the sphere, the Bloch ball. This representation has many convenient geometric properties.

Here we propose a generalization of this Bloch sphere representation to arbitrary Hilbert space dimension. Our approach provides a compact and elegant representation of spin density matrices in terms of tensors that share the most important properties of Bloch vectors. This representation is based on covariant matrices introduced by Weinberg in the context of quantum field theory, and allows for a simple parametrization of coherent spin states. We will start with an introduction to this new tensor representation. Then we will show how to use it to construct matrices that provide entanglement criteria similar to the partial transpose criteria for entanglement of multiqubit quantum states. Finally, we apply the recently introduced formalism of tensor eigenvalues and eigenvectors in order to investigate to what extent tensor eigenvalues contain information about multipartite entanglement of the state. In particular we show that there is a correlation between entanglement and the value of the smallest tensor eigenvalue.

Network Clustering With Higher Order Structures

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Abstract. Spectral clustering is a well-known way to partition a graph or network into clusters or communities with provable guarantees on the quality of the clusters. This guarantee is known as the Cheeger inequality and it holds for undirected graphs. We'll discuss a new generalization of the Cheeger inequality to higher-order structures in networks including network motifs. This is easy to implement and seamlessly generalizes spectral clustering to directed, signed, and many other types of complex networks. In particular, our generalization allow us to re-use the large history of existing ideas in spectral clustering including local methods, overlapping methods, and relationships with kernel k-means. We will illustrate the types of clusters or communities found by our new method in biological, neuroscience, ecological, transportation, and social networks.

Homotopy Continuation Methods in Tensor Computations

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Abstract. Nonnegative tensors and M-tensors have found applications in various areas, such as high order Markov chains, spectral hypergraph theory, statistical data analysis, numerical solution of partial differential equations, and tensor complementarity problems.

We consider two computational problems involving M-tensors and nonnegative tensors. The first problem is to find the unique positive solution to a multilinear system with a nonsingular M-tensor and a positive right side vector, and the second one is to compute the largest eigenvalue of an irreducible nonnegative tensor. Recently, homotopy continuation methods have been proposed to solve these problems. In this talk, I will present these homotopy methods. The talk consists of two parts.

In the first part, I will describe a homotopy method for finding the unique positive solution to a multilinear system with a nonsingular M-tensor and a positive right side vector. The method is proved to converge to the desired solution. In the second part, I will introduce a homotopy method for computing the largest eigenvalue and a corresponding eigenvector of a nonnegative tensor. The convergence to the desired eigenpair is guaranteed when the tensor is irreducible. Both homotopy methods have been implemented using an prediction-correction approach for path following. I will present some numerical results to illustrate the effectiveness of these methods.

Part of this talk is based on joint work with Liping Chen, Hongxia Yin, and Liangmin Zhou.

Inverse Eigenvalue Problem for Tensors

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Abstract. Let $\mathbb{T}(\mathbb{C}^n, m+1)$ be the space of tensors of order $m+1$ and dimension n with complex entries. A tensor $\mathcal{T} \in \mathbb{T}(\mathbb{C}^n, m+1)$ has nm^{n-1} eigenvalues (counted with algebraic multiplicities). The inverse eigenvalue problem for tensors is a generalization of the inverse eigenvalue problem for matrices. Namely, given a multiset $S \in \mathbb{C}^{nm^{n-1}}/\mathfrak{S}(nm^{n-1})$ of total multiplicity nm^{n-1}, is there a tensor in $\mathbb{T}(\mathbb{C}^n, m+1)$ such that the set of eigenvalues of $\mathcal{T}$ is exactly S? The solvability of the inverse eigenvalue problem for tensors is studied in this article. With tools from algebraic geometry, it is proved that the necessary and sufficient condition for this inverse problem to be generically solvable is $m = 1,$ or $n = 2,$ or $(n, m) = (3, 2),\ (4, 2),\ (3, 3).$

This is a joint work with Ke Ye

Tensor Variational Inequalities

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Abstract. In this talk, we introduce a class of variational inequalities, where the involved function is the sum of an arbitrary given vector and a homogeneous polynomial defined by a tensor; and we call it the tensor variational inequality (TVI). The TVI is a natural extension of the affine variational inequality and the tensor complementarity problem. We show that a class of multi-person noncooperative games can be formulated as a TVI. In particular, we investigate the global uniqueness and solvability of the TVI. To this end, we first introduce two classes of structured tensors and discuss some related properties; and then, we show that the TVI has the property of global uniqueness and solvability under some assumptions, which is different from the existed result for the general variational inequality.

Sparse Interpolation Via Super-resolution, Compressed Sensing and Prony

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Abstract. We show that solving the sparse polynomial (multivariate) interpolation problem reduces to solving a discrete super-resolution problem on the n-dimensional Torus. Therefore the semidefinite programming approach initiated by Cands & Fernandez-Granda [7] in the univariate case (and later extended to the multivariate setting) can be can be applied. In particular, for n=1,2, exact recovery is guaranteed provided that a geometric spacing condition on the support" holds and the number of evaluations are sufficiently many (but not many). It also turns out that under the same spacing condition, exact recovery is also guaranteed via the (compressed sensing) LP-formulation of l1-norm minimization provided that evaluations are made in a certain manner and even though the RIP sufficient condition is not satisfied ; a naive compressed sensing LP-approach does no offer such a guarantee. Finally we also discuss this approach with the algebraic Prony method which for n=1 requires twice as less point evaluations and no geometric spacing condition but seems to be less robust to noise in the data.

The Sparse Non-negative Tensor Equations Arriving From Data Sciences

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Abstract. In this talk, we consider nonnegative tenor equations arriving from the data analysis. The uniqueness of the solution for the equations can be obtained. We also give some variations of the solution when the coefficient tensor is perturbed. Numerical examples are given to illustrate that theoretical results are efficient.
Joint work with DD Liu, M. Ng and SW Vong

Tensor Network Ranks

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Abstract. A universal problem in science and engineering is to find a function from some input data. The function may be a solution to a PDE and with given boundary/initial data or a target function to be learned from a training set of data. In modern applications, one frequently encounters situations where the function lies in some state space or hypothesis space of prohibitively high dimension — a consequence of, say, very high accuracy solution or massive training set. A common remedy with newfound popularity is to assume that the function has low rank, i.e., may be expressed as a sum of a small number of separable terms. But this is often a bad assumption with weak justification. We will define a vast generalization of the classical notions of rank (i.e., matrix rank, tensor rank, multilinear rank) — given any undirected graph G, there is a G-rank associated with that graph. In particular, the wildly popular tensor network states in physics (e.g., MPS, TTNS, PEPS, MERA) are nothing more than functions of a fixed G-rank for different choices of G. We will discuss various properties of G-ranks. For instance, we will see that a function may have (arbitrarily) high tensor rank and yet (arbitrarily) low G-rank for some G. This is joint work with Ke Ye.

Cone Eigenvalue Complementarity Problems for Higher-Order Tensors and Some Polynomial Optimization Models

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Abstract. In this talk, we consider the tensor generalized eigenvalue complementarity problem (TGEiCP) and tensor higher-degree eigenvalue complementarity problem (THDEiCP), which are generalizations of eigenvalue complementarity problem (EiCP) and quadratic eigenvalue complementarity problem (QEiCP) of matrices, respectively. First, we show that the considered problems are solvable and have at least one solution under some reasonable assumptions. Then, we introduce some optimization reformulations of TGEiCP and THDEiCP. Based on these, upper bounds of cone eigenvalues and the number of eigenvalues are discussed. Two implementable algorithms for solving TGEiCP and THDEiCP are developed for the problem under consideration, and some preliminary computational results are also reported.

The Sparsest Solutions to Z-Tensor Complementarity Problems

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Abstract. Finding the sparsest solutions to a tensor complementarity problem is generally NP-hard due to the non-convexity and discontinuity of the involved ell_0 norm. In this talk, a special type of tensor complementarity problems with Z-tensors has been considered. Under some mild conditions, we show that to pursuit the sparsest solutions is equivalent to solving polynomial programming with a linear objective function. The involved conditions guarantee the desired exact relaxation and also allow to achieve a global optimal solution to the relaxed non-convex polynomial programming problem. Particularly, in comparison to existing exact relaxation conditions, such as RIP-type ones, our proposed conditions are easy to verify.

Infinite Dimensional Hilbert Tensors on Spaces of Analytic Functions

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Abstract. In this paper, the $m-$order infinite dimensional Hilbert tensor (hypermatrix) is intrduced to define an $(m-1)$-homogeneous operator on the spaces of analytic functions, which is called Hilbert tensor operator. The boundedness of Hilbert tensor operator is presented on Bergman spaces A^p $(p > 2(m-1))$. On the base of the boundedness, two positively homogeneous operators are introduced to the spaces of analytic functions, and hence the upper bounds of norm of such two operators are found on Bergman spaces A^p $(p > 2(m-1))$. In particular, the norms of such two operators on Bergman spaces $A^{4(m-1)}$ are smaller than or equal to π and $\pi^{\frac{1}{m-1}}$, respectively.
This work joins with Professor Liqun Qi

Neural Networks for Computing Best Rank-One Approximations of Tensors and its Applications

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Abstract. This talk presents the neural dynamic network to compute a best rank-one approximation of a real-valued tensor. We implement the neural network model by the ordinary differential equation (ODE), which is a class of continuous-time recurrent neural network. Several properties of solutions for the neural network are established. We prove that the locally asymptotic stability of solutions for ODE by establishing an appropriate Lyapunov function under mild conditions. Furthermore, we also consider how to use the proposal neural networks for solving the tensor eigenvalue problem including the tensor H-eigenvalue problem, the tensor Z-eigenvalue problem, and the generalized eigenvalue problem with symmetric-de nite tensor pairs. Finally, we gener- alize the proposal neural networks to the computation of the restricted singular values and the associated restricted singular vectors of real-valued tensors. We illustrate theoretical results via numerical examples.
Joint with Maolin Che and A. Cichocki

Two New Definitions of Eigenvalues for Fourth-order Tensor and Some Properties

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Abstract. In this talk, I introduce two new definitions of eigenvalues for fourth-order tensor. These new definitions may be regarded as the extensions of existing definitions for tensor in the fourth-order situation. We show some relationships between these different notions. And we also define two classes of positivity semi-definite tensors and give some connections between non-negativity of new tensor eigenvalues and positive semi-definiteness. Based on them, we present an algorithm solving the new defined largest eigenvalue and a procedure finding the existing largest eigenvalue for the symmetric tensors. Finally we report some numerical examples.

A Fast Algorithm for the Spectral Radius for Symmetric Nonnegative Tensors

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Abstract. In this talk, a fast algorithm for computing the spectral radius for symmetric nonnegative tensors is presented. Numerical results are reported to show that the proposed algorithm is efficient.

Contributed Talks

Moment Fusion Based Methods for Gaussian Mixture Estimation

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Abstract. We propose a novel data fusion based method **DF-GMM** for Gaussian mixture model (GMM) estimation. Fusion of third-order (tensor), second-order (matrix) and first-order (vector) moments, makes the GMM parameters essentially unique. We use L-BFGS to optimize the above problem and study the performance of **DF-GMM** for different choices of the moment weights. As an alternative approach, we consider a prewhitening-based algorithm **CPD-GMM**. The prewhitening turns the problem into an orthogonal symmetric canonical polyadic decomposition (CPD) of a third-order tensor. The prewhitening highly improves the accuracy and significantly reduces the computational time. Numerical experiments confirm our findings.

Octupolar Tensors for Liquid Crystals

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Abstract. To describe tetrahedratic nematic phases in a molecular assembly of liquid crystals, a third-order three dimensional symmetric traceless tensor, named the octupolar tensor, was studied. The octupolar potential, a scalar-valued function of that tensor on a unit sphere, would ideally have four maxima (a tetrahedron) capturing most probable molecular orientations. It was shown that the admissible region of the octupolar tensor is under a surface called the dome and there exists a separatrix surface connecting ideally generic octupolar states and distorted ones. Such a separatrix surface may physically represent an intra-octupolar transition. In this talk, by using the resultant theory in algebraic geometry and the E-characteristic polynomial in spectral theory of tensors, we give the algebraic expressions of the dome and the separatrix surface explicitly. This may turn such an intra-octupolar transition an observable prediction. Some other properties of octupolar tensors are also studied.

A Quadratic Penalty Method for Hypergraph Matching

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Abstract. Hypergraph matching is a fundamental problem in computer vision. Mathematically speaking, it maximizes a polynomial objective function, subject to assignment constraints. In this paper, we reformulate the hypergraph matching problem as a sparse constrained tensor optimization problem. The optimality conditions are characterized based on the sparse constrained optimization theory. By dropping the sparsity constraint, we show that the resulting relaxation problem can recover the global minimizer of the original problem. The key step in solving the original problem is to identify the location of nonzero entries (referred as support set) in a global minimizer. Inspired by such observations, we penalize the equality constraints and apply the quadratic penalty method to solve the relaxation problem. Under reasonable assumptions, we show that the support set of the global minimizer in hypergraph matching problem can be correctly identified when the number of iterations is sufficiently large. A projected gradient method is applied as a subsolver to solve the quadratic penalty subproblem. Numerical results demonstrate that the exact recovery of support set indeed happens, and the proposed algorithms are efficient in terms of both accuracy and speed.

Hermitian Tensors and Their Properties

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Abstract. In quantum physics, a m-partite pure state $|\psi\rangle$ of a composite quantum system can be regarded as a normalized element in a Hilbert tensor product space $\mathbb{C}^{n_1\times\cdots\times n_m}$. The pure state $|\psi\rangle$ is denoted as $|\psi\rangle = \sum_{i_1,\cdots,i_m=1}^{n_1,\cdots,n_m} \chi_{i_1\cdots i_m}|e_{i_1}^{(1)}\cdots e_{i_m}^{(m)}\rangle$, where $\chi_{i_1\cdots i_m}\in\mathbb{C}$, $\{|e_{i_k}^{(k)}\rangle : i_k = 1,2,\cdots,n_k\}$ is an orthonormal basis of $\mathbb{C}^{n_k}$. Hence, a pure state is uniquely corresponding to a complex tensor $\mathcal{X} = (\mathcal{X}_{i_1\cdots i_m})$ under a given orthonormal basis. For a general mixed state ρ, its density matrix is always written as $\rho = \sum_{i=1}^{k}\lambda_i|\psi_i\rangle\langle\psi_i|$, where $\sum_{i=1}^{k}\lambda_i = 1$, $|\psi_i\rangle$ is a pure state and $\langle\psi_i|$ is the complex conjugate transpose of $|\psi_i\rangle$. Hence, the density matrix of ρ is also uniquely corresponding to a Hermitian tensor $\mathcal{H}\in\mathbb{C}^{n_1\times\cdots\times n_m\times n_1\times\cdots\times n_m}$ with $\mathcal{H}_{i_1\ldots i_m j_1\ldots j_m} = \mathcal{H}^*_{j_1\ldots j_m i_1\ldots i_m}$ under a given orthonormal basis. In quantum physics, there are many properties on the separable problem and entanglement value problem of quantum mixed states. All these properties can also be seen as properties of Hermitian tensors. In this talk, we introduce the concept of Hermitian tensors, rank-one Hermitian decomposition, unitary equivalent relationship, and invariant properties under unitary transformation. Meanwhile, we use tensor methods to study properties of quantum mixed states, including Schmidt polar form, criterion for multipartite separable states and entanglement values, etc.

Third Order Tensors and Hypermatrices

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Abstract. Third order tensors have wide applications in mechanics, physics and engineering. The most famous and useful third order tensor is the piezoelectric tensor, which plays a key role in the piezoelectric effect, first discovered by Curie brothers. On the other hand, the Levi-Civita tensor is famous in tensor calculus. In this paper, we study third order tensors and (third order) hypermatrices systematically, by regarding a third order tensor as a linear operator which transforms a second order tensor to a first order tensor, and a first order tensor to a second order tensor. We introduce the transpose, the kernel tensor and the inverse of a third order tensor. The transpose of a third order tensor is uniquely defined. The kernel tensor of a third order tensor is a second order positive semi-definite symmetric tensor, which is the product of that third order tensor and its transpose. We define non-singularity for a third order tensor. A third order tensor has an inverse if and only if it is nonsingular. We also define eigenvalues, singular values, C-eigenvalues and Z-eigenvalues for a third order tensor. They are all invariants of that third order tensor. A third order tensor is nonsingular if and only if all of its eigenvalues are positive. Physical meanings of these new concepts are discussed. We show that the Levi-Civita tensor is nonsingular, its inverse is a half of itself, and its three eigenvalues are all the square root of two. We also introduce third order orthogonal tensors. Third order orthogonal tensors are nonsingular. Their inverses are their transposes.

Brualdi-type Singular Value Inclusion Sets of Rectangular Tensors

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Abstract. In this paper, by using digraphs of rectangular tensors, we give Brualdi-type singular value inclusion sets of rectangular tensors. Then based on the relationship between the singular values of rectangular tensor and the eigenvalues of related lifting square tensor, the weakly irreducibility of lifting square tensor are presented, thus we extend some results of tensors to rectangular tensors. Finally, we give an example to show that the singular value inclusion sets of rectangular tensors given by using digraphs is better than the results given by the known eigenvalue inclusion sets of lifting square tensors.

Quantum Entanglement and Tensor Eigenvalues

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Abstract. The geometric measure of quantum entanglement for pure states has attracted much attention. On the other hand, the spectral theory of non-negative tensors has been developed rapidly. In the first part of this talk, we show how the spectral theory of non-negative tensors can be applied to the study of the geometric measure of entanglement for pure states. In the second part of this talk, we discuss how to use tensors to determine whether a spin-j state is quantum or classical, more specifically, we show that a spin-j state is classical if and only if its representing tensor is a regularly decomposable tensor.

Stream 5

Statistical Modeling and Computational Statistics

Invited Talks

Monitoring Test by Empirical Likelihood and Resampling Method

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Abstract. The reliability of a wood structure depends on the strength of lumber product from the market. Closely monitoring the dynamic wood strength distribution of the solid lumber over the time is therefore an important quality control task. Due to the potential catastrophic event could be induced by a few weak wood piece, the lower quantiles of the strength distribution received most attention in the subject discipline. Clearly, monitoring only a single lower quantile of the wood strength is not sufficient. Even if the strength distribution of the wood product meets the quality standard specified on lower quantiles, the median or mean strengths could be significantly lower than the norm. In this presentation, we combine empirical likelihood and resampling method to construct a monitoring test for multiple quality indices. The proposed test tightly controls the type I error and is sensitive at detecting the quality reduction.

Efficient Feature Screening for Ultrahigh-dimensional Varying Coefficient Models

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Abstract. Feature screening in ultrahigh-dimensional varying coefficient models is a crucial statistical problem in economics, genomics and etc. Existing methods suffer in the cases of multiple index variables and group predictor variables. Moreover, current methods cannot handle nonlinear varying coefficient models which are possible in reality. To deal with those scenarios efficiently in real life, we develop a screening procedure for ultrahigh-dimensional varying coefficient models utilizing conditional distance covariance (CDC). Extensive simulation studies and two real economic data examples have shown the effectiveness and the flexibility of our proposed methods.

Two-Layer Heterogeneity Model for Massive Data

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Abstract. Massive data generally consist of numerous heterogeneous datasets, while some of them may be thought of as being from the same sub-population due to their similarity. We attempt to capture this subtle but important data structure by using a Two-layer HEterogeneity Model (THEM) framework that accounts for heterogeneity among sub-populations and within sub-populations. Furthermore, a confidence distribution fusion approach is proposed to discover the latent sub-population structure. This big data analysis tool can be easily implemented in a parallel fashion through ADMM. Moreover, the statistical inferential accuracy is proven to achieve its highest level as if the true sub-population structure were known. In the end, the proposed methodology is applied to a big climate dataset that reveals a possible association with the El Nino-Southern Oscillation.
This is a joint work with Ching-Wei Cheng at Purdue.

Strategies to Facilitate Access to Detailed Geocoding Information Based on Synthetic Data

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Abstract. Results are presented on whether generating synthetic data can be a viable strategy to provide access to detailed geocoding information for external researchers, without compromising the confidentiality of the units included in the database. This research was motivated by a recent project at the Institute for Employment Research (IAB) in Germany that linked exact geocodes to the Integrated Employment Biographies, a large administrative database containing several million records. Based on these data, we evaluate the performance of several synthesizers in terms of addressing the trade-of between preserving analytical validity and limiting the risk of disclosure. We propose strategies for making the synthesizers scalable for such large files, introduce analytical validity measures for the generated data, and provide general recommendations for statistical agencies considering the synthetic data approach for disseminating detailed geographical information

Bayesian Emulation for Multi-step Portfolio Decisions

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Abstract. In the context of sequential forecasting and portfolio optimization, we introduce a new approach to Bayesian analysis based on mapping a specified loss function minimization problem to that of finding the mode of a posterior distribution in a synthetic statistical model. Computational methods for exploring distributions can then be applied to solve the original optimization problem. We do this in the context of novel portfolio utility functions that extend traditional Markowitz-type methods to multiple-step ahead investments with explicit penalties for transaction costs. Various forms introduced and explored include sparsity-inducing penalties on portfolio turnover and asymmetric penalties on the deviation from the target return, which yield interesting classes of synthetic statistical models of state-space forms with non-Gaussian structure. The resulting computational problems are addressed using combinations of EM, MCMC and analytic filtering and smoothing. Significant practical benefits in application to financial portfolio are realized in applied studies of FX, commodity and stock index time series, based on sequential forecasting using customized dynamic dependency network models.

A Functional Varying-Coefficient Single-Index Model for Functional Response Data

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Abstract. Motivated by the analysis of imaging data, we propose a novel functional varying-coefficient single-index model (FVCSIM) to carry out the regression analysis of functional response data on a set of covariates of interest. FVCSIM represents a new extension of varying-coefficient single-index models for scalar responses collected from cross-sectional and longitudinal studies. An efficient estimation procedure is developed to iteratively estimate varying coefficient functions, link functions, index parameter vectors, and the covariance function of individual functions.We systematically examine the asymptotic properties of all estimators including theweak convergence of the estimated varying coefficient functions, the asymptotic distribution of the estimated index parameter vectors, and the uniform convergence rate of the estimated covariance function and their spectrum. Simulation studies are carried out to assess the finite-sample performance of the proposed procedure. We apply FVCSIM to investigate the development of white matter diffusivities along the corpus callosum skeleton obtained from Alzheimers Disease Neuroimaging Initiative (ADNI) study. Supplementary material for this article is available online.

Distributed Semi-supervised Learning

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Abstract. DSKRR applies kernel ridge regression (KRR) to each data subset that is distributively stored on multiple servers to produce an output function, and then takes a weighted average of the individual output functions as a final estimator. Our results show that unlabeled data play important roles in enlarging the number of data subsets in DSKRR. Optimal learning rates are established for DSKRR in the framework of learning theory. Numerical experiments including toy simulations and a music-prediction task are employed to demonstrate our theoretical statements and show the power of unlabeled data in distributed learning.

A Look at Computing, Modeling, and Inference with Examples

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Abstract. In this talk, we take a look at three problems in Data Science: (1) computing with SupR, (2) penalty or regularization, and (3) deep learning in artificial intelligence and statistical modeling. We start the talk with a brief introduction to SupR, an experimental software for big data analysis, which has a R-style front-end, a Java-like multithreading model, an Apache Spark-like cluster computing environment, and a built-in Simple Distributed File System. The penalty-based approach has become an indispensable tool in frequentist statistics. From the perspective of prior-free probabilistic inference, with the Stein paradox as an illustrative example, we discuss what penalty function is. We conclude this talk with a few remarks on an understanding of the development of artificial intelligence and recent success of deep learning and on how such an understanding could help statistical modeling with big data.

i-Fusion: Efficient Fusion Learning Method for Individualized Inference from Diverse Sources

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Abstract. Inferences from different databases or studies can often be fused together to yield a more powerful overall inference than individual studies alone. Fusion learning refers to effective approaches for such synergizing learnings from different data sources. Effective fusion learning is of vital importance, especially in light of the ubiquitous massive automatic data collection nowadays. Decision-making processes in many domains such as medicine, life science, social studies, etc. can benefit greatly from fusion learning from different sources, possibly with varying forms of complexity and heterogeneity in data structure.

This talk presents some new fusion methodologies for extracting and merging useful information. Particular focus is the i-Fusion method, which a novel learning by individual-to-clique approach to fuse information from relevant entities to make inference for the target individual entity. Drawing inference from a clique allows borrowing strength from similar entities to enhance the inference efficiency for each individual. The i-Fusion method is flexible and computationally efficient. It can also be scaled up to search through massive database with complex structure. The key tool underlying those fusion methodologies is the so-called confidence distribution (CD), which, simply put, is a versatile distributional inferential scheme (unlike the usual point or interval inferences) without priors. Time permits, applications of the i-Fusion method in financial modeling, star formation in galaxies, precision medicine and weather forecast will also be discussed.

This is joint work with John Kolassa, Jieli Shen and Minge Xie, Rutgers University.

Individualized Multilayer Learning with An Application in Breast Cancer Imaging

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Abstract. This work is motivated by breast cancer imaging data produced by a multimodal multiphoton optical imaging technique. One unique aspect of breast cancer imaging is that different individuals might have breast imaging at different locations, which also creates a technical difficulty in that the imaging background could vary for different individuals. We develop a multilayer tensor learning method to predict disease status effectively through utilizing subject-wise imaging information. In particular, we construct an individualized multilayer model which leverages an additional layer of individual structure of imaging in addition to employing a high-order tensor decomposition shared by populations. In addition, to incorporate multimodal imaging data for different profiling of tissue, cellular and molecular levels, we propose a higher order tensor representation to combine multiple sources of information at different modalities, so important features associated with disease status and clinical outcomes can be extracted effectively. One major advantage of our approach is that we are able to capture the spatial information of microvesicles observed in certain modalities of optical imaging through combining multimodal imaging data. This has medical and clinical significance since microvesicles are more frequently observed among cancer patients than healthy ones, and identification of microvesicles enables us to provide an effective diagnostic tool for early-stage cancer detection.
This is joint work with Xiwei Tang and Xuan Bi.

Analysis of Generalized Semiparametric Mixed Varying-Coefficient Effects Model for Longitudinal Data

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Abstract. The generalized semiparametric mixed varying-coefficient effects model for longitudinal data that can flexibly model different types of covariate effects. Different link functions can be selected to provide a rich family of models for longitudinal data. The mixed varying-coefficient effects model accommodates constant effects, time-varying effects, and covariate-varying effects. The time-varying effects are unspecified functions of time and the covariate-varying effects are nonparametric functions of a possibly time-dependent exposure variable. We develop the semiparametric estimation procedure by using local linear smoothing and profile weighted least squares estimation techniques. The method requires smoothing in two different and yet connected domains for time and the time-dependent exposure variable. The estimators of the nonparametric effects are obtained through aggregations to improve efficiency. The asymptotic properties are investigated for the estimators of both nonparametric and parametric effects. Some hypothesis tests are developed to examine the covariate effects. The finite sample properties of the proposed estimators and tests are examined through simulations with satisfactory performances. The proposed methods are used to analyze the ACTG 244 clinical trial to investigate the effects of antiretroviral treatment switching in HIV infected patients before and after developing the codon 215 mutation.

Spatial Quantile Regression Models for High-dimensional Imaging Data

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Abstract. The aim of this paper is to develop a spatial quantile regression framework for accurately quantifying high-dimensional image data conditional on some scalar predictors. This new framework allows us to delineate spatial quantile association between neuroimaging data and covariates, while explicitly modeling spatial dependence in neuroimaging data. Theoretically, we establish the minimax rates of convergence for the prediction risk under both fixed and random designs. We further develop efficient algorithms such as the ADMM and the primal-dual algorithm to estimate the varying coefficients. Our method is able to estimate the whole conditional distribution of the image response given the scalar covariates. Simulations and real data analysis are conducted to examine the finite-sample performance.

This is a joint work with Zhengwu Zhang, Linglong Kong, and Hongtu Zhu.

Likelihood Inference for a Continuous Time GARCH Model

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Abstract. The continuous time GARCH (COGARCH) model of Kluppelberg, Lindner and Maller (2004) is a natural extension of the discrete time GARCH(1,1) model which preserves important features of the GARCH model in the discrete-time setting. For example, the COGARCH model is driven by a single source of noise as in the discrete time GARCH model, which is a Levy process in the COGARCH case, and both models can produced heavy tailed marginal returns even when the driving noise is light-tailed. However, calibrating the COGARCH model to data is a challenge, especially when observations of the COGARCH process are obtained at irregularly spaced time points. The method of moments has had some success in the case with regularly spaced data, yet it is not clear how to make it work in the more interesting case with irregularly spaced data. As a well-known method of estimation, the maximum likelihood method has not been developed for the COGARCH model, even in the quite simple case with the driving Levy process being compound Poisson, though a quasi-maximum likelihood (QML) method has been proposed. The challenge with the maximum likelihood method in this context is mainly due to the lack of a tractable form for the likelihood. In this talk, we propose a Monte Carlo method to approximate the likelihood of the compound Poisson driven COGARCH model. We evaluate the performance of the resulting maximum likelihood (ML) estimator using simulated data, and illustrate its application with high frequency exchange rate data.

Estimating The Spectrum Of A High Dimensional Covariance Matrix Via Anticommuting Variables

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Abstract. Let S_p be a $p \times p$ sample covariance matrix such that nS_p has the Wishart distribution $W_p(n, \Sigma_p)$ where Σ_p is a $p \times p$ population covariance matrix. Knowing S_p, we propose a class of estimators for the spectrum of Σ_p. The estimators are derived by essentially minimizing the distance between the empirical Stieltjes transform of S_p and its expectation. The latter is expressed as a double integral over $(0, \infty)^2$ by the supersymmetry method involving Grassmann or anticommuting variables. Under suitable conditions, these estimators are shown to be weakly consistent as $p \to \infty$ such that p/n tends to some constant $c > 0$. Simulations indicate that the proposed estimators perform well relative to other state-of-the-art spectrum estimators.

Overlapping Sliced Inverse Regression for Dimension Reduction

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Abstract. Sliced inverse regression(SIR) is statistical tool for dimension reduction. It identifies the effective dimension reduction space, the subspace of significant factors with intrinsic lower dimensionality. We propose refined implementations of SIR algorithm by allowing slice overlapping. The new algorithms, called overlapping-enabled sliced inverse regression (OSIR), can estimate the effective dimension reduction space and determine the number of effective factors more accurately. We show that the overlapping technique codes the information of the differences (or derivatives in the limit sense) of the inverse regression curve, which helps to explain the superiority of OSIR. We also prove that OSIR algorithms are $\sqrt{n}$-consistent and verifie the effectiveness of OSIR algorithms by simulations and applications.

Nonparametric Shrinkage Estimation

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Abstract. We revisit the classical problem of estimating the mean vector of three or more observations. James and Stein (1961) claimed that given a known upper bound for the fourth moment of the noise, a shrinkage estimator of Stein (1956) always has a smaller total mean squared error than the observed vector itself as a naive estimator, provided that certain parameters of the shrinkage factor are properly specified. James and Stein (1961) further commented that “It would be desirable to obtain explicit formulas for estimators one can seriously recommend” in this setting. In the present paper, we provide some explicit solutions to this nonparametric shrinkage estimation problem.

Estimating Network Edge Probabilities by Neighborhood Smoothing

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Abstract. The problem of estimating probabilities of network edges from the observed adjacency matrix has important applications to predicting missing links and network denoising. It has usually been addressed by estimating the graphon, a function that determines the matrix of edge probabilities, but is ill-defined without strong assumptions on the network structure. Here we propose a novel computationally efficient method based on neighborhood smoothing to estimate the expectation of the adjacency matrix directly, without making the strong structural assumptions graphon estimation requires. The neighborhood smoothing method requires little tuning, has a competitive mean-squared error rate, and outperforms many benchmark methods on the task of link prediction in both simulated and real networks.

Some New Understandings of Tree-based Methods in Survival Analysis

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Abstract. As one of the most popular machine learning tools, tree-based method has been adapted for survival analysis and estimate the conditional survival functions nonparametrically. We investigate the method from the aspect of splitting rules, where the log-rank test statistics is calculated and compared to find the best splitting variable. We demonstrate that this approach is affected by the censoring distributions, which may lead to inconsistency of the method. Based on this observation, we develop an adaptive concentration bound in the sense that for each terminal node, the estimation centers around the true within node average of the underlying survival model, which could be affected by the censoring distribution. As a result, we show that the consistency of the method by can be achieved if the splitting rule is modified to satisfy certain restrictions.

Contributed Talks

A New Measuring Method of the Gini Coefficient and the Empirical Analysis

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Abstract. Residents' income distribution is related to the general public living standards. The Gini coefficient is an important index for measuring income distribution. This paper discusses a new method of estimating the Gini coefficient. By using the China statistical yearbook data from 2002-2011, the new method is used to estimate Lorenz curves, and then estimate the Gini coefficient. The new model is established, such as

$$L_1(p,v) = p^{\alpha}\{\delta[1 - L_{\lambda}(1-p,w)^{\beta}] + (1-\delta)L_{\lambda_1}(p,w)\}^{\eta}.$$

Using Mathematical software, we estimate the parameter vector $v = (\alpha, \beta, \eta, \delta, w, \lambda, \lambda_1)$. Since our country urban and rural income distribution data are grouped data, we should respectively calculate Gini coefficients. Then we measure the Gini coefficient of our country by the formula

$$G = p_1^2 \frac{u_1}{u} G_1 + p_2^2 \frac{u_2}{u} G_2 + p_1 p_2 \left| \frac{u_2 - u_1}{u} \right|.$$

The results show that China's Gini coefficient increases year by year and has passed the "red line". The Gini coefficients of urban and rural areas are increased but still under 0.4 and relatively reasonable distribution. So the main reason of our country's higher Gini coefficient is that the income gap between town and country is larger.

Improvement Screening for Ultra-High Dimensional Data with Censored Survival Outcomes and Varying Coefficients

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Abstract. Motivated by risk prediction studies with ultra-high dimensional biomarkers, we propose a novel improvement screening methodology. Accurate risk prediction can be quite useful for patient treatment selection, prevention strategy or disease management in evidence-based medicine. The question of how to choose new markers in addition to the conventional ones is especially important. In the past decade, a number of new measures for quantifying the added value from the new markers were proposed, among which the integrated discrimination improvement (IDI) and net reclassification improvement (NRI) stand out. Meanwhile, C-statistics are routinely used to quantify the capacity of the estimated risk score in discriminating among subjects with different event times. In this paper, we will examine these improvement statistics as well as the norm-based approach for evaluating the incremental values of new markers and compare these four measures by analyzing ultra-high dimensional censored survival data. In particular, we consider Cox proportional hazards models with varying coefficients. All measures perform very well in simulations and we illustrate our methods in an application to a lung cancer study.

Keywords: Risk prediction; Diagnostic accuracy improvement; Integrated discrimination improvement; Net reclassification improvement; C-statistics; l^2-norm

Stream 6
Financial Mathematics

Invited Talks

Capital Accumulation and Real Options

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Abstract. We study here the situation of a firm which exploits an external resource, and decides its investments at appropriate times, in the spirit of real options. However, we are interested in a sequence of projects, and not just a single one. Each project represents a substantial investment, with fixed cost and variable costs measuring the scale of the project. At the same time, the firm is growing and thus accumulates capital, which puts it each time in a more favorable position to exploit the external resource. The problem is to define the sequence of optimal stopping times to invest. We follow the methodology of impulse control, in which the value function is the solution of a Quasi Variational Inequality (QVI). We obtain new types of QVI, which we can solve in some particular cases.

A Primal-Dual Iterative Monte Carlo Method for Stochastic Dynamic Programs and Its Applications in Finance

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Abstract. In this paper we use the information relaxation technique to develop a value-and-policy iterative method to solve stochastic dynamic programming problems. Each iteration generates a confidence interval estimate for the true value function and a corresponding suboptimal policy so that we can use the gap between the upper and lower bounds to access the quality of the policy. We show that the resulted sequences of suboptimal policies will converge to the optimal one within finite number of iterations through our method.

A regression-based Monte Carlo algorithm is introduced to overcome the dimensionality curse in the implementation of this approach for high dimensional cases. Our formulation reduces the original problem to solving a sequence of open loop control problems. We can thereby rely on a variety of well-developed deterministic optimization algorithms to accelerate the computational speed. It is different from the traditional literature of approximate dynamic programs where a majority of methods need to solve stochastic optimization problems. As numerical illustrations, we apply the algorithm to the optimal order execution problem and the portfolio selection problems. Some new insights about optimal value and optimal policy are also discussed.

On Optimal Pricing Model for Multiple Dealers in a Competitive Market

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Abstract. In this talk, we discuss optimal pricing strategy for a monopolistic dealer in a general situation where multiple dealers are present in a competitive market. The dealers' trading intensities, their optimal bid and ask prices and therefore their spreads are derived when the dealers are informed the severity of the competition. The effects of various parameters on the bid-ask quotes and profits of the dealers in a competitive market are also discussed. This study gives some insights on the average spread, profit of the dealers in a competitive trading environment.

General Dynamic Term Structures under Default Risk

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Abstract. We consider the problem of modeling the term structure of bonds subject to default risk, under minimal assumptions on the default time. In particular, we do not assume the existence of a default intensity and we therefore allow for the possibility of default at predictable times. It turns out that, in order to exclude arbitrage possibilities, this requires the introduction of an additional term to the forward rate approach by Heath, Jarrow and Morton (1992). This term is driven by a random measure encoding information about those times where default can happen with positive probability. In this framework, we derive necessary and sufficient conditions for a reference probability measure to be a local martingale measure for the large financial market of credit risky bonds, also considering general recovery schemes.
This is based on joint work with Thorsten Schmidt (University of Freiburg).

Over-the-Counter Markets with Counterparty Risk

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Abstract. We develop a parsimonious model for over-the-counter (OTC) markets, where trading decisions depend on counterparty risk. Market participants are risk-averse banks exposed to a risky loan portfolio. Before trading begins, each bank makes a costly decision on its risk management policies, which affect its default risk. Afterwards, traders choose their counterparties and trading volumes. The market trades OTC derivatives similar to credit default swaps (CDS). We show that the equilibrium outcomes reproduce stylized factors of the structure of OTC markets. Banks with relatively low default risk share their risk from the loan portfolio so that they all have similar post-trade exposures. In contrast, riskier banks have diverse post-trade exposures because of the asymmetry in the risk profile of CDS buyers and sellers. In fact, riskier banks are less attractive as sellers while as buyers their default risk is less relevant to their counterparties. The model predicts that safe banks with intermediate initial exposure emerge as dealers in this market. The predictions of the model are tested using data of the CDS market.

Rough Volatility and Related Topics

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Abstract. The rough volatility model has recently attracted much attention due to its consistency to stylized facts of both historical and implied volatility data. The volatility of an asset is driven by a fractional Brownian motion which is correlated to a Brownian motion driving the asset. By rough we mean that the Hurst parameter is less than half. In this talk, we discuss several recent advances (most of which are ongoing works) including the short-term at-the-money asymptotics of the implied volatility and the high-frequency data analysis of fractional Gaussian noises with some empirical evidences.

An Optimal Consumption, Gift, Investment, and Voluntary Retirement Choice Problem with Quadratic and HARA Utility

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Abstract. We study an optimal consumption, gift, investment, and voluntary retirement choice model of an agent who has a motive for giving by using a utility function. The utility function in the objective function is given by the weighted sum of a quadratic utility function and a HARA utility function. We use the martingale method to derive a closed form solution for optimal consumption, gift and investment. We also give some numerical implications.

Markovwitz Portfolio and the Blur of History

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Abstract. Due to the phenomenon of "blur of history", the estimation efficiency of the mean annual rate of returns cannot be improved by increasing the sampling frequency. In this talk, it is illustrated that estimation of mean return vector can be avoided in the construction of Markovwitz optimal portfolio. To achieve this goal, several assumptions are imposed: (i) the asset returns are generated from a factor model, (ii) the number of assets goes to infinity, and (iii) there is no asymptotic arbitrage opportunity. Issues related to the efficiency of the estimated optimal portfolio for high-frequency data are discussed. The portfolio constructed using estimated mean vector and those using a factor model are compared.

Optimality of Hybrid Continuous and Periodic Barrier Strategies in the Dual Model

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Abstract. Avanzi et al. (2016) recently studied the optimal dividend problem where dividends can be paid both periodically and continuously at different transaction costs. In the Brownian model with Poissonian periodic dividend payment opportunities, they showed that the optimal strategy is of pure-continuous, pure-periodic or hybrid-barrier-type. In this paper, we generalize their results to the dual (spectrally positive Lévy) model. The optimal strategy is again of hybrid-barrier-type and can be concisely expressed using the scale function. The results are confirmed through a sequence of numerical experiments.

Zero-sum Stochastic Differential Games Without the Isaacs Condition:random Rules of Priority and Intermediate Hamiltonians

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Abstract. For a zero-sum stochastic game which does not satisfy the Isaacs condition, we provide a value function representation for an Isaacs-type equation whose Hamiltonian lies in between the lower and upper Hamiltonians, as a convex combination of the two. For the general case (i.e. the convex combination is time and state dependent) our representation amounts to a random change of the rules of the game, to allow each player at any moment to see the other player's action or not, according to a coin toss with probabilities of heads and tails given by the convex combination appearing in the PDE. If the combination is state independent, then the rules can be set all in advance, in a deterministic way. This means that tossing the coin along the game, or tossing it repeatedly right at the beginning leads to the same value. The representations are asymptotic, over time discretizations. Space discretization is possible as well, leading to similar results. The talk is based on joint work with Daniel Hernandez-Hernandez.

A Self-Exciting Threshold Jump-Diffusion Model for Option Valuation

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Abstract. In this talk, we shall discuss a self-exciting threshold jump-diffusion model for option valuation. This model incorporates regime switches without introducing an exogenous stochastic factor process. A generalized version of the Esscher transform is adopted to select a pricing kernel. The valuation of both the European and American contingent claims is considered. A piecewise linear partial differential-integral equation governing a price of a standard European contingent claim is obtained. For an American contingent claim, a formula decomposing a price of the American claim into the sum of its European counterpart and the early exercise premium is provided. An approximate solution to the early exercise premium based on the quadratic approximation technique is obtained for a particular case where the jump component is absent. Some numerical results for European and American options are presented for the case without jumps.

Duality for American Options in Non-dominated Discrete-time Models

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Abstract. The classical pricing-hedging duality for American options with semi-static hedging does not hold in general in the simple formulation inherited from European option set-up. We propose two approaches to recover the duality result. The first approach consists in considering a bigger class of models and rendering an American option a European one. The second way is to relax the static trading and by allowing dynamic trading in the set of vanilla options. As a by-product, it is proved that the dynamic trading of vanilla options does change the optimal pricing-hedging value comparing to the semi-static trading strategy. The connections to classical enlargement of filtration are also discussed. The problem of stopping only at stopping times w.r.t. price process filtration can be related to pseudo-stopping times (as well randomized stopping times) and the immersion property.
Based on a joint work with Anna Aksamit, Shuoqing Deng, and Jan Obłój.

Pricing Real Call and Put Options of European and American Types

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Abstract. In this talk we present our recent advances in developing PDE-based mathematical models for valuing real options on investment project expansions and contraction, as well as project ownership transfer, when the underlying asset price follows a geometric Brownian motion. The models developed are of a similar form as the Black-Scholes model for pricing conventional European options or the linear complementarity problem for valuing conventional American options, but, unlike the standard models,, the payoff conditions are determined by a PDE system. A finite volume scheme coupled with a penalty method is used for solving the models and a convergence analysis is performed for the penalty approach. Numerical experiments have been performed using model problems in pricing options on project expansion, contraction and ownership transfer in the mining industry to demonstrate that our models and numerical method are able to produce financially meaningful numerical results for the non-trivial test problems.

Persistence and Procyclicality in Margin Requirements

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Abstract. Margin requirements for derivative contracts serve as a buffer against the transmission of losses through the financial system by protecting one party to a contract against default by the other party. However, if margin levels are proportional to volatility, then a spike in volatility leads to potentially destabilizing margin calls in times of market stress. Risk-sensitive margin requirements are thus procyclical in the sense that they amplify shocks. We use a GARCH model of volatility and a combination of theoretical and empirical results to analyze how much higher margin levels need to be to avoid procyclicality while reducing counterparty credit risk. Our analysis compares the tail decay of conditional and unconditional loss distributions to compare stable and risk-sensitive margin requirements. Greater persistence and burstiness in volatility leads to a slower decay in the tail of the unconditional distribution and a higher buffer needed to avoid procyclicality. The tail decay drives other measures of procyclicality as well. Our analysis points to important features of price time series that should inform "anti-procyclicality" measures but are missing from current rules.
This is a joint work with Paul Glasserman.

A Time-inconsistent Stochastic Optimal Control Problem With Recursive Cost Functional

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Abstract. This talk is concerned with an optimal control problem with the state equation being a stochastic differential equation and with the cost functional being a recursive cost functional described by a backward stochastic differential equation. Also, a general discount is included in the cost functional, due to which, the problem becomes time-inconsistent. By using multi-person differential game approach, we derived an equilibrium Hamilton-Jacobi-Bellman equation, through which, an equilibrium strategy could be constructed.
This work is joint with Qingmeng Wei and Zhiyong Yu.

Ambiguity Aversion and Optimal Derivative-based Pension Investment With Stochastic Income and Volatility

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Abstract. This paper provides a derivative-based optimal investment strategy for an ambiguity averse pension investor who faces not only risks from time-varying income and market return volatility but also uncertainty of economic condition over a long-time horizon. We derive a robust dynamic derivative strategy and show that the optimal strategy under ambiguity aversion reduces the exposure to market return risk while increases the exposure to stochastic volatility risk, while derivatives can effectively hedge stochastic volatility risk. More importantly, we demonstrate the welfare improvement when considering ambiguity and exploiting derivatives and show that ambiguity aversion and derivatives improve the welfare significantly when return volatility increases, while the improvement becomes more significant under ambiguity aversion over long investment time horizon.

Dynamic Portfolio Optimization With Looping Contagion Risk

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Abstract. Dynamic portfolio optimization with looping contagion risk We consider a dynamic portfolio optimization problem in a financial market with equities, credit derivatives and contagion risk. We assume a looping contagion model in which the default intensity of one company depends on its counterparty's stock price, while the default on the other way round induces an immediate drop in its counterparty's stock price. By separating the utility maximization problem into a pre-default and post-default component, we deduce two coupled HJB equations and show that the pre-default value function is the unique constrained viscosity solution to the associated HJB equation. We obtain explicit solutions of value functions and trading strategies for investors with log utility, and show by a numerical test that the contagion risk has large impact on investor's optimal trading strategy. For power utility investors, we express the optimal trading strategy as the unique solution of an equation system and prove that the value function is the unique viscosity solution associated with a BSDE.
Joint work with Longjie Jia and Martijn Pistorius

Optimal Management of Crash Risk

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Abstract. Crash of a financial market means that most of the financial assets suddenly lose a certain part of their nominal value. When a crash happens, almost all the asset returns become perfectly correlated, the diversification effect of portfolios in a normal market condition does not work any longer and the traditional risk measures largely underestimate the risk. Thus, for portfolio management, the performance measures (risk and return) and the managerial point of view under a crash should be distinguished from the traditional one under a normal market condition. In this work, we integrate crash risk into portfolio management and investigate the performance measures, hedging and optimization of portfolio selection involving crash risk. A convex conic programming framework based on parametric approximation method is proposed to formulate the problem as a tractable one. Comprehensive simulation analysis and empirical study are performed to test the proposed approach.
This is a joint work with Wei Zhu, Xi Pei and Xueting Cui.

Contributed Talks

Optimal Consumption, Portfolio, and Life Insurance Under Luxury Bequest Motive and Inflation Risk

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Abstract. In this talk, we study an optimal consumption and portfolio selection problem of a breadwinner with a mandatory retirement date T>0 who has luxury bequest motives and faces the non-negative bequest constraint under inflation risk. During the working period of the breadwinner, the family purchases life insurance to protect an absence of labor income caused by sudden death of breadwinner In this case, we obtain a closed-form solution by using a martingale approach, and investigate the effects of inflation risk on the consumption/investment strategies.

Workshop 1

Numerical Methods for Phase-field models

Invited Talks

The Application of Parallel, Adaptive, Multigrid to Phase-field Simulation

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Abstract. This presentation will focus on our study of efficient numerical algorithms for the solutions of general phase-field models. The unifying theme to our work is an interest in problems which feature a wide range of time scales, and are therefore require implicit, stiff, solvers for their time integration. The talk will be broken down into three parts as follows. Firstly, the development and application of an efficient, mesh-adaptive, multigrid framework will be described for 3-d phase-field problems. This is based upon an implicit multistep time integration scheme and a block-structured adaptive discretization in space. At each time-step a nonlinear algebraic system is solved using a nonlinear multigrid solver based upon the MLAT approach. The talk will then show two different examples of the successful application of the software that we have developed: one using a model of the non-isothermal rapid solidification of a binary metallic alloy and the other using a multiphase model for tumour growth. Finally, the presentation will conclude with a discussion of the parallel implementation issues associated with combining multigrid with mesh adaptivity. The resulting dynamic load-balancing problem will be described and the effectiveness of a number of different dynamic load-balancing algorithms will be considered.

Unconditional Stability of Semi-implicit Methods in Phase Field Models

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Abstract. Recent results in the literature provide computational evidence that stabilized semi-implicit time-stepping method can efficiently simulate phase field problems involving fourth-order nonlinear dif- fusion, with typical examples like the Cahn-Hilliard equation and the thin film type equation. The up-to-date theoretical explanation of the numerical stability relies on the assumption that the deriva- tive of the nonlinear potential function satisfies a Lipschitz type condition, which in a rigorous sense, implies the boundedness of the numerical solution. I will discuss a group of recent results which remove the Lipschitz assumption on the nonlinearity and prove unconditional energy stability for the stabilized semi-implicit time-stepping methods.
This is joint work with Tao Tang and Zhonghua Qiao

An Augmented Adi Method for Heat Equations With a Discontinuous Coefficient and Singular Source Along an Arbitrary Interface

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Abstract. A new second-order accurate ADI (alternating direction implicit) method for heat equations with a piecewise constant but discontinuous coefficient and a singular source along an arbitrary but fixed interface has been developed. In the literature, an earlier ADI method by Z. Li & A. Mayo for heat equations with an interface assuming constant coefficient was developed. While the ADI method developed by S. Zhao in JSC for interface problems works for discontinuous coefficients but seems to be only first order accurate in time and the resulting linear system of equation is pentadiagonal instead of tridiagonal in the original ADI method. Two novel new ideas make the new ADI working possible. The first idea is to reformulate, or preconditioning the partial differential equation so that the discontinuous coefficient is not involved in the highest derivative term. The second idea is to introduce an augmented variable along the interface to get the dimension-by-dimension jump conditions needed to get accurate discretization in coordinate direction.

The developed ADI method is second order accurate and conditionally stable with a favorable time step constrain $k\backslash le$ C, where the constant C depends on the coefficient.

Diffusion in Biological Environments

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Abstract. Recently we had been working on projects related to charge transport in solutions and proteins (ion channels). One of the key ingredients in these studies is the understanding of diffusion and its relations to other effects, such as hydrodynamics, electrostatics and other particle-particle interactions. Due to the non-ideal situations in almost all biological environments, such as the high concentration of charge densities, those conventional theories have to be modified or re-derived.

In the talk, I will employ the general framework of energetic variational approaches, especially Onsager's Maximum Dissipation Principles to the problems of generalized diffusion. We will discuss the roles of different stochastic integrations, and the procedures of optimal transport in the context of general linear response theory in statistical physics.

Competitive Evolution of Multicomponent Amphiphilic Bilayer Networks

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Abstract. Many biological applications, in particular the endoplasmic reticulum, are comprised of a collections of amphiphilic lipids that encode distinct morphological preferences. We present a fourth order free energy whose mass-preserving gradient flows engender a center-stable reduction that couples geometric flow of the underlying membrane shape to phase separation of lipid types upon the surface of the membrane. We discuss issues that arise with numerical simulation of network morphologies that support high-frequency modulations, describe applications to physical systems, and present an analysis of a minimal model that high-lights the complexity of the underlying dynamics. These latter including a possible infinite dimensional 'quenched flow' that represents a slow motion to long-wave periodic modulations of bilayer composition.

Phase Field Modeling and Simulations for Non-isothermal One-component Liquid-vapor Systems

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Abstract. The dynamic van der Waals theory has been presented by Akira Onuki for one-component fluids, capable of describing the two-phase hydrodynamics involving liquid-vapor transition in non-uniform temperature field. It has been employed to model and simulate the growth of a single vapor bubble in a superheated liquid on a heated substrate. The bubble spreading dynamics has been numerically investigated for pool boiling. It is seen that for near critical fluids, the thermal singularity at moving contact line plays a key role in heat transfer and liquid-vapor transition.

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A New Class of Linear, Decoupled, Energy Stable Schemes for Gradient Flows

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Abstract. We present in this talk a new way to construct linear, decoupled, energy stable schemes for gradient flows. The new class of schemes enjoy the following advantages: (i) at each time step, one only has to solve decoupled, linear positive definite systems with *constant coefficients*; (ii) it is proved to be unconditionally energy stable and appears to be quantitatively more accurate than existing schemes of the same order; (iii) it applies to a wider class of problems.

Phase-field Study of Hydride Morphology in Zirconium

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Abstract. Zirconium and its alloys are key structural materials used in the nuclear power industry. In service, these metals are susceptible to a slow corrosion process that leads to a gradual pickup of hydrogen impurities from the environment. It is well known that hydrogen impurity will migrate under stress and/or temperature gradients. At a certain hydrogen level, a complicated pattern of hydride precipitates can develop. Because of the brittleness of these hydrides, the original strength of the alloys can be reduced by orders of magnitude, and the fracture through these hydrides may occur. It is believed that critical conditions for the initiation of fractures at hydrides are controlled by the morphology and microstructure of hydride precipitates. In recent years, the authors research team has developed a phase-field scheme to simulate the morphological and microstructural evolution of hydride precipitation in single and polycrystalline zirconium under uniform and non-uniform stress and temperature fields. Recent effort was devoted to develop a quantitative model. The model takes into account crystallographic variants of hydrides, interfacial energy between hydride and matrix, interfacial energy between hydrides, elastoplastic hydride precipitation and interaction with externally applied stress and/or temperature field. The model is fully quantitative in real time and real length scale, and simulation results were compared with limited experimental data available in the literature with a reasonable agreement. This work was supported by grants from Research Grants Council of Hong Kong (PolyU 5267/10E) and the National Natural Science Foundation of China (No.51271157).

Energy Stable Algorithms for Diffuse Interface Modeling of Partially Miscible Two-Phase Systems

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Abstract. Fluid mixture involving two or multiple phases are important and common systems in petroleum industry, where oil, gas and water are often produced and transported together. Petroleum reservoir engineers spent great efforts in drainage problems arising from the development and production of oil and gas reservoirs so as to obtain a high economic recovery, by developing, conducting, and interpolating the simulation of subsurface flows of reservoir fluids, including water, hydrocarbon, CO2, H2S for example in porous geological formation. Field-scale or Darcy-scale simulation has conventionally and routinely used for this purpose, but this approach requires a number of poorly known parameters including relative permeability and capillary pressure. Pore-scale simulation of two-phase systems, however, can lead to information on these parameters as well as to provide deep understanding of porous media flow and transport.

In this work, we consider two-phase flow with partial miscibility at a pore scale. Specifically, we study the modeling and simulation of possibly compressible, partially miscible, fully compositional two-phase hydrocarbon systems using a diffuse interface model together with Peng-Robinson Equation of State (EOS). This research has an eventual goal of applying to realistic modeling of petroleum and other reservoir fluids in pores or pore networks within geological formation. Our modeling scheme utilizes molar densities as the order parameters, and the approach is based on the coupling of the Navier-Stokes equation for flow and a Cahn-Hilliard-like equation with Peng-Robinson chemical potentials for phase behaviors of hydrocarbon fluids. Our modeling approach can be used to predict volumetric behaviors, solubility, miscibility, and interface tensions of common hydrocarbon liquid (oil) and vapor (gas) accurately. Moreover, the entire modeling approach is self-consistent and complies with the principles of non-equilibrium thermodynamics including the second law of thermodynamics and the Onsager reciprocity principle. To solve the continuum model expressed as a coupled nonlinear partial differential equation (PDE) system, we propose an efficient numerical solution of the modeling system, focusing on discrete energy stability, local mass conservation and numerical accuracy. For spatial discretization, we apply a finite volume-based method to turn the partial differential equations (PDE) into an ordinary differential equation (ODE) system. For temporal discretization, the resultant ODE system is decoupled by using an asymmetric splitting scheme, and then integrated in time using a semi-implicit marching scheme. In addition, targeting the specific features of each of the three terms in Peng-Robinson chemical potentials, we propose a convex splitting-based semi-implicit time scheme, which is proved to

be unconditionally energy stable under certain conditions. We also proposed an energy-stable component-wise splitting to treat the multi-species system efficiently. We compare our computational results with laboratory experimental data reported in the literature, which have good agreement.

This presentation is partially based on joint work with Xiaolin Fan (KAUST), Jisheng Kou (Hubei Eng. U.), Yiteng Li (KAUST), Zhonghua Qiao (HK PolyU), and Tao Zhang (KAUST).

Complex Dewetting Scenarios of Ultra-thin Films for Large-scale Nano-architectures

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Abstract. Owing to their large surface-area-to-volume ratio, thin solid films are often unstable upon annealing. Under the action of surface diffusion the film breaks eventually forming isolated islands. This is one of the main factors impeding the use of ultra-thin silicon films on insulators (UT-SOI) for the further miniaturization of electronic components. Furthermore, the poor spatial organization and large size dispersion of the islands render this self-assembly approach unsuitable for other applications such as optical metasurfaces. With a lithographic method, the ultimate control of UT-SOI dewetting for the precise formation of complex nano-architectures featuring extremely reduced fluctuations of size, shape and positioning (a few of repetitions and on large scales could be demonstrated. Predictive phase-field simulations of the mass transport mechanism, assess the dominant role of surface diffusion providing a tool for further engineering this hybrid top-down/bottom-up self-assembly method. We provide details on the modeling and the numerical approach which enables this large scale simulations.

Generalized Onsager Principle and the Development of Numerical Schemes

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Abstract. Onsager principle is a paradigm for deriving governing equations for nonequilibrium systems, especially, for general thermodynamics and generalized hydrodynamics of dissipative properties. It can be extended to yield the generalized Onsager principle that can be used to derive a wider class of nonequilibrium theories. In the generalized Onsager principle, variational and the traditional Onsager principle is effectively integrated to yield the extended nonequilibrium theoretical framework. The models derived using the generalized Onsager principle take up a large portion of the known theories for materials, especially, flowing materials. In this presentation, I will show how one can exploit the mathematical structure of the models derived using the generalized Onsager principle to develop energy stable numerical schemes. I will illustrate the idea using a few examples from multiphasic fluid flows and complex fluid flows.

Preconditioned Steepest Descent (PSD) Solver for Regularized P-laplacian Equations

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Abstract. A few preconditioned steepest descent (PSD) solvers are presented for the fourth and sixth-order nonlinear elliptic equations that include p-Laplacian terms. The highest and lowest order terms are constant-coefficient, positive linear operators. Instead of solving the nonlinear systems directly, we minimize the convex energies associated with the the equations. By using the energy dissipation property, we derive a discrete bound for the solution, as well as an upper-bound for the second derivative of the energy. These bounds allow us to investigate the convergence properties of our method. In particular, a geometric convergence rate is shown for the nonlinear PSD iteration applied to the regularized equation, which provides a much sharper theoretical result over the existing works. Some numerical simulation results are also presented in the talk, such as the thin film epitaxy with both $p = 4$ and $p = 6$, as well as the gradient flow of the squared phase field crystal (SPFC) model.

An Energy Stable Decoupled Scheme for the Navier-Stokes-Darcy Flow

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Abstract. Many applications involve the coupling of free flow with flow in porous media. The governing equation is the Navier-Stokes-Darcy system which enjoys an energy law under appropriate interface boundary conditions. It is well-known that the numerical simulation of the system is a challenge due to the multi-scale (in space and in time) nature of the problem. In this talk, I present a novel numerical algorithm that inherits the energy law of the Navier-Stokes-Darcy system. Moreover, the scheme is decoupled in the sense that only a Navier-Stokes solver and a Darcy solver are needed at each time step. So far as we know, this is the first algorithm that enjoys the energy stability and decouples the Navier-Stokes and Darcy solver for the Navier-Stokes-Darcy system.

a General Framework for High Accuracy Solutions to Energy Gradient Flows From Material Science Models

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Abstract. A computational framework is presented for phase field materials science models that come from energy gradient flows. The framework includes higher order derivative models and vector problems. Solutions are considered in periodic cells and standard Fourier spectral discretization in space is used. Fully implicit time stepping is used with adaptivity based on local error estimates. The implicit system at every time step is solved iteratively with Newton's iterations, with the linear systems solved at each step with a preconditioned conjugate gradient method. Solutions with high spatial and temporal accuracy are obtained. It is shown that large fully implicit time steps can be taken in times of meta-stable dynamics. A comparison to time-stepping with operator splitting (into convex and concave parts that guarantees energy decrease in the numerical scheme) is done. It is shown numerically and analytically how these schemes can be inaccurate through meta-stable dynamics.

A Novel Numerical Approach to Solve a Class of Nonlinear Gradient Flow Models

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Abstract. The nonlinear thermodynamically consistent model is usually derived from the variational approach of the free energy. Such modeling approaches are commonly used in almost all physical passive system that satisfies the laws of thermo-mechanics. One of the main numerical challenges is about the development of the time marching scheme with stability, efficiency and higher order accuracy. We introduce a novel, so called Invariant Energy Quadratization approach, that can possess many desired properties. More precisely, the schemes (i) are accurate (up to second order in time); (ii) are stable (unconditional energy dissipation law holds); and iii) only need to solve a linear, symmetric positive definite system at each time step.

Invariant Energy Quadrtization Approach for Incompressible Smectic-A Liquid Crystal Flow

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Abstract. In this talk, we construct the first-order decoupled and the second-order coupled temporal accurate energy stable schemes to solve the smectic-A liquid crystals model in the incompressible fluid. This model involves the incompressible Navier-Stokes equation and a fourth order phase-field equation for the order parameter of smectic-A liquid crystals. The projection method is adopted to decouple the pressure and the velocity in the Navier-Stokes equation. Moreover, the Invariant Energy Quadratization method is used to linear the nonlinear functionals, meanwhile preserving the energy stability. Then we present the fully discrete energy stable schemes by using finite differences in time and C0-finite elements in space where the fourth order equation shall be transformed into two second order equations. At last we calculate several numerical tests to illustrate the temporal accuracy of the proposed schemes and verify that our schemes are unconditionally energy stable. Moreover, we simulate the process of undulation in the smectic-A liquid crystals under the effect of the shear flows and the magnetic field, which is consistent with other works.

Phase Field Simulations of Electrocaloric Effect in Perovskite Ferroelectrics

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Abstract. The electrocaloric effect of ferroelectric materials, which occurs significantly near the first-order paraelectric/ferroelectric transition Curie temperature, has tremendous prospect in solid-state cooling devices. In the present work, thermodynamics analysis and phase field simulations were conducted to demonstrate the mechanical compression-induced two types of pseudo-first-order phase transition, which could occur at a temperature below the Curie temperature. Thus, in one material there may coexist ultrahigh positive and negative electrocaloric effects, which are associated with the two pseudo-first-order phase transitions and tunable by the magnitude of the compression. The mechanical compression-induced pseudo-first-order phase transition and the coexistence of positive and negative electrocaloric effects establish a novel technology to design and manufacture next generation of solid-state cooling devices.

The simulations were conducted by Dr. Hong-Hui Wu during his PhD study at Hong Kong University of Science and Technology.

A Second Order Scheme and Numerical Analysis for the Three-dimensional Phase Field Crystal Equation

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Abstract. We analyze and implement a second-order-in-time numerical scheme for the three dimensional phase field crystal (PFC) equation. The numerical scheme has been proposed with the unique solvability and unconditional energy stability established. However, its convergence analysis remains open. We present a detailed convergence analysis in this work, in which the maximum norm estimate of the numerical solution over grid points plays an essential role. Moreover, we outline the detailed multigrid method to solve this highly nonlinear numerical scheme over a three-dimensional domain, and various numerical results are presented, including the numerical convergence test, complexity test of the multigrid solver and the polycrystal growth simulation.

Contributed Talks

First- and Second-Order Stabilized Semi-Implicit Schemes for the Nonlocal Cahn-Hilliard Equation

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Abstract. In this talk, we present the numerical analysis on the stabilized schemes for the nonlocal Cahn-Hilliard (nCH) equation. The nCH equation is the H^{-1} gradient flow with respect to the nonlocal energy functional coming from the classical Ginzburg-Landau free energy with the Laplacian replaced by a well-defined nonlocal diffusion operator. The lack of the high order diffusion term becomes the main difficulty for the numerical analysis and the existence of the nonlocal term usually leads to expensive calculations in the numerical simulation. Thus, efficient and accurate numerical schemes preserving the energy stability are highly desired. We propose the stabilized semi-implicit schemes with first- and second-order temporal accuracy for the nCH equation with periodic boundary condition, by treating the nonlocal diffusion term implicitly while nonlinear chemical potential part explicitly and adding an artificial term for the sake of stability. The energy stabilities and error estimates of both schemes are rigorously established in the time-discrete sense. Numerical experiments are presented to verify the convergence rates of the proposed schemes, make a comparison of the phase transition process with the corresponding local case, and simulate the coarsening dynamics to predict the $-1/3$ power law of the energy decay rate.

Numerical solutions for a Phase Field Model with Peng-Robinson Equation of State

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Abstract. This work is concerned with mathematical modeling and numerical simulations of the steady state and the movements of complex fluids involved in oil exploitation practice. Capillary pressure caused by surface tension at the interface between every two adjacent different phases of the mixture is viewed as the leading force in oil recovery from fractured oil reservoirs. Therefore, the interface between contiguous phases has become a critical mathematical modeling aspect. The phase field model has been widely applied to model or understand the interface between different phases of oil mixture. Based on the assumption that the density of every substance is continuous over the whole fluid region, the total Helmholtz free energy often contains the homogeneous part and the gradient contribution part. Based on the original total free energy, the equilibrium state and the kinetic processes could be determined according to thermodynamic principles. As for the fluid system related to the oil recovery process, we apply the homogeneous free energy density and the parameters of the gradient part of the free energy density provided by the widely used Peng-Robinson equation of state (EOS). The gradient flow equation based on this energy functional are presented and solved by several energy stable schemes. The theoretical analyses of these numerical schemes and numerical results demonstrate good properties of these numerical schemes.

A Mass Conserving Lattice Boltzmann Scheme for 3D Diffuse Interface Model With Peng-Robinson Equation of State

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Abstract. In this work, a realistic 3D diffuse intercface model with Peng-Robinson equation of state (EOS) for two phase fluid systems are numerical studied by the lattice Boltzmann (LB) method. Based on the gradient theory of thermodynamics and variational calculus, a generalized chemical equilibrium equation with extremely strong nonlinear source term is obtained. Through adding a time-derivative term, the steady-state equation is converted to a time-dependent parabolic partial differential equation (PDE). In addition, a Lagrange multiplier is introduced to enforce mass conservation. And then, a mass conserving LB scheme is proposed to solve this transient system. The definition of the Lagrange multiplier is based on the mesoscopic character of the proposed LB scheme. Furthermore, through the multi-scale Chapman-Enskog analysis, the time-dependent parabolic PDE can be recovered with second order accuracy from the proposed LB scheme. Finally, the 3D numerical simulations of realistic hydrocarbon components, such as isobutane and propane, are implemented to illustrate the theory. The numerical results have a good agreement with laboratory data through considering the width of the two phase interface.

Phase-field-based Lattice Boltzmann Model for Thermocapillary Flows

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Abstract. In this paper, a phase-field-based lattice Boltzmann (LB) model is proposed for thermocapillary flows with variable fluid-property ratios. In this model, three lattice Boltzmann equations are used to describe the flow, phase and temperature fields. Compared to the available LB models for thermocapillary flows, it can be shown that through the Chapman-Enskog analysis, the macroscopic governing equation can be recovered correctly from present LB model. We also test the present LB model through two classical problems, i.e., the thermocapillary driven convection in two superimposed fluids and thermocapillary migration of a deformable droplet, and find that the present results are in agreement with some available theoretical and numerical results.

Workshop 2

Big Data Analysis with Applications

Invited Talks

Exact Robust Counterparts of Ambiguous Stochastic Constraints Under Mean and Dispersion Information

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Abstract. In this talk we will study optimization problems with ambiguous stochastic constraints where only partial information consisting of means and dispersion measures of the underlying random parameter is available. Whereas the past literature used the variance as the dispersion measure, here we use the mean absolute deviation from the mean (MAD). The approach is based on the availability of tight upper and lower bounds on the expectation of a convex function of a random variable, first discovered in 1972. We then use these bounds to derive exact robust counterparts of expected feasibility of convex constraints and to construct new safe tractable approximations of chance constraints. We test the applicability of the theoretical results numerically on various practical problems in Operations Research and Engineering

Data-Driven Analysis of Human Dynamics Using Wi-Fi

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Abstract. Human dynamics refer to movement and interactions of large crowd. Analysis of human dynamics is important for both social science and computer science. It enables a wide range of applications in many areas like marketing, management, and sociology. For example, understanding customers shopping behaviors is vital for retailers to adapt their marketing strategies. However, studying human dynamics requires massive data, which is non-trivial to collect. Previous research mainly focuses on phone call records collected by network operators to study coarse-grained human activities. Nowadays, Wi-Fi-enabled smartphones are increasingly ubiquitous throughout the world and open up a great opportunity to collect massive human activity data in an effective and efficient way. On the other hand, however, it also raises new challenges due to vulnerability and sparsity of wireless signal. Besides, how to extract effective features from Wi-Fi packets still remains an open challenge. In this talk, I will introduce recent advances in human dynamics and then discuss opportunities and challenges followed by future research directions. I will also describe our recent work on analysis of human dynamics using Wi-Fi.

Statistical Inference with Stochastic Gradient Descent

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Abstract. In this talk, we investigate the problem of statistical inference of the true model parameters based on stochastic gradient descent (SGD). To this end, we propose two consistent estimators of the asymptotic covariance of the average iterate from SGD: (1) an intuitive plug-in estimator and (2) a computationally more efficient batch-means estimator, which only uses the iterates from SGD. As the SGD process forms a time-inhomogeneous Markov chain, our batch-means estimator with carefully chosen increasing batch sizes generalizes the classical batch-means estimator designed for time-homogenous Markov chains. Both proposed estimators allow us to construct asymptotically exact confidence intervals and hypothesis tests. We further discuss an extension to conducting inference based on SGD for high-dimensional linear regression.

Large Scale Data Problems in Data Assimilation

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Abstract. Data assimilation covers techniques where prediction of the state of a dynamical systems is performed using data from various origins. We consider here the optimization problem that lies in the centre of this technique, when so-called variational formulations are considered. Our main interest will be focused on case where the dynamical systems under consideration is described by *stochastic* differential equations. Such a problem is called "Weak-constrained variational Data Assimilation".

We will presents the merits of the three main approaches that are considered by the community: state, forcing, saddle-point formulations. All three formulations lead to a sequence of large scale linear systems which must be solved iteratively. The crucial ingredient to reach computational efficiency is the design of an efficient preconditioner in each approach. In this talk, we will first consider the use of Quasi-Newton formula to design preconditioners, focusing mostly on a the saddle-point approach in which a form of sparsity of the dynamical system can be taken into account.

Numerical experiments are discussed that compare several preconditioning techniques on an instructional example. This illustrates the challenges of the approach and also its potential whose assessment is ongoing.

Optimization Models and Algorithms for Fingerprint Recognition

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Abstract. In this talk, a general optimization model of automatic fingerprint identification system (AFIS) is proposed. For solving the general model, a serial of optimization models and algorithms are established and designed, including the module of feature extraction and minutiae matching in AFIS. We proposed a global optimization model for orientation field computation, variable dimension optimization for singular point detection, and bipartite graph optimization model for minutiae matching. According to the characteristics of fingerprint image, novel algorithms are designed for these three models. These algorithms were embedded in our AFIS, which has been successfully applied to many provinces (cities) in China. It has played an important role in cracking and preventing all kinds of criminal cases.

The Space and Time of Neuronal Variability in a Spiking Neuron Network

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Abstract. Shared variability among neurons (noise correlations) have been commonly observed in multiple cortical areas (Cohen and Kohn, 2011). Moreover, noise correlations are modulated by cognitive factors, such as overall arousal, task engagement and attention (Cohen and Maunsell, 2009; Doiron et al. 2016). While there is much discussion about the consequences of noise correlations on neuronal coding, there is a general lack of understanding of the circuit mechanisms that generate and modulate shared variability in the brain. Recently, simultaneous microelectrode array recordings from V1 and MT in behaving monkeys (Ruff and Cohen, 2016) suggest that attention not only decreases correlations within a cortical area (MT), but also increases correlations between cortical areas (V1 and MT). The differential modulation of between-areas and within-area noise correlations impose further constraints on circuit mechanisms for the generation and propagation of noise correlations. We develop a spiking neuron network with spatiotemporal dynamics that internally generates shared variability matching the low dimensional structure widely reported across cortex. This variability results from macroscopic chaos in population rates, which correlates neurons from the same recurrent network while decoupling them from feedforward inputs. Attention is modeled as depolarizing the inhibitory neuron population, which reduces the internally generated shared variability and allows the network to better track input signal. Moreover, dimensionality analysis of spike trains in both model and data shows that attention modulates only the first dimension of fluctuations, again consistent with past phenomenological models of cortical data. Our model provides a much needed mechanism for how shared variability is both generated and modulated in recurrent cortical networks.

Deep Learning for Semantic Text Matching

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Abstract. Semantic matching is critical in many text applications, including paraphrase identification, information retrieval, question answering, and machine translation. A variety of machine learning techniques have been developed for various semantic matching tasks, referred to as learning to match. Recently, deep learning approaches have shown their effectiveness in this area, and a number of methods have been proposed from different aspects of matching. In this talk, I will give a systematic survey on newly developed deep learning technologies for semantic matching. I will focus on the descriptions on the fundamental problems, as well as the novel solutions from bridging the word level semantic gap and conducting sentence level end-to-end semantic matching. I will also discuss the potential applications and future directions of semantic matching for text.

Spatially Clustered Coefficient Regression Models for Large Data Sets

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Abstract. The availability of large spatial and spatial-temporal data geocoded at accurate locations has fueled increasing interest in spatial modeling and analysis. Geographically Weighted Regression (GWR) and Spatially-Varying Coefficients (SVC) model are two popular methods used to explore the spatially varying relationship between response variable and predictors. Both methods explicitly or implicitly assume stationarity in the spatially varying coefficient surface over space, making them unsuitable for cases in which coefficients exhibit cluster structure with homogeneity within clusters and abrupt changes across the boundary of clusters. In this article, we propose a spatially clustered varying coefficient regression model via regularization to better detect the spatially abruptly changing effect of explanatory variables. Oracle inequalities are developed to provide non-asymptotic error bounds on the estimation and prediction. The utility of the proposed method is also demonstrated in simulation studies and a real example with temperature and salinity data in the Atlantic basin.

Learn-and-Adapt Stochastic Dual Gradients for Network Resource Allocation

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Abstract. Network resource allocation shows revived popularity in the era of data deluge and information explosion. Existing stochastic optimization approaches fall short in attaining a desirable cost-delay tradeoff. Recognizing the central role of Lagrange multipliers in network resource allocation, a novel learn-and-adapt stochastic dual gradient (LA-SDG) method is developed in this paper to learn the empirical optimal Lagrange multiplier from historical data, and adapt to the upcoming resource allocation strategy. Remarkably, it only requires one more sample (gradient) evaluation than the celebrated stochastic dual gradient (SDG) method. LA-SDG can be interpreted as a foresighted learning approach with an eye on the future, or, a modified heavy-ball approach from an optimization viewpoint. It is established both theoretically and empirically - that LA-SDG markedly improves the cost-delay tradeoff over state-of-the-art allocation schemes.

A Parallel Approach for Orthogonal Constrained Optimization Problem

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Abstract. To construct parallel approach for solving orthogonal constrained optimization problems is usually regarded as an impossible mission, due to the low scalability of orthogonalization procedure. In this talk, we propose a Jacobi type column-wise block coordinate descent method for solving differentiable orthogonal constrained optimization problems, and establish the global iterate convergence to stationary point of our proposed approach. Preliminary numerical experiments illustrate the efficiency of our new algorithm under both in serial and parallel settings. Especially, our new algorithm exhibits high scalability in solving discretized Kohn-Sham total energy minimization problems.

Geometric Descent Method for Convex Composite Minimization

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Abstract. We extend the geometric descent method recently proposed by Bubeck, Lee and Singh to tackle nonsmooth and strongly convex composite problems. We prove that our proposed algorithm, dubbed geometric proximal gradient method (GeoPG), converges with a linear rate $(1-1/\sqrt{\kappa})$, and thus achieves the optimal rate among first-order methods, where κ is the condition number of the problem. Numerical results on linear regression and logistic regression with elastic net regularization show that GeoPG compares favorably with Nesterov's accelerated proximal gradient method, especially when the problem is ill-conditioned.

An Evolving Subspace Method for Low Rank Minimization

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Abstract. I present a method for solving low rank minimization problems that combines subspace minimization techniques, inexact subspace conditions to terminate exploration of the subspace, and inexact singular value decompositions. Taking together, these features allow the algorithm to scale well, and in fact be competitive with nonconvex approaches that are often used. Convergence results are discussed and preliminary numerical experiments are provided.

A Framework for Globally Convergent Methods in Nonsmooth and Nonconvex Problems

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Abstract. Large scale nonsmooth and nonconvex optimization models arise in various data science paradigms. They induce many highly challenging mathematical and computational issues. In this talk, we outline a fairly general theoretical framework to derive globally convergent schemes. We then show how this framework can be successfully applied and adapted to design and analyze novel algorithms for various classes of non smooth and nonconvex models, by exploiting friendly structures and data information of the problem at hands. Numerical results will illustrate our findings.

On the Statistical Performance and Convergence Rate of the Generalized Power Method for Angular Synchronization

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Abstract. The problem of angular synchronization with its numerous applications has attracted intense research recent years. Motivated by its order-optimal estimation error in the sense of Cramer-Rao inequality, two approaches have been recently developed to compute the maximum likelihood estimator. One is the semidefinite relaxation and the other is to directly solve the non-convex maximum likelihood optimization by the generalized power method (GPM). In this talk, we focus on the latter approach and study it from both statistical and optimization-theoretic point of view. First, we take a new perspective by directly studying the estimation error of GPM and show that the l2- and l 1-error bounds decrease geometrically to the best provable bound under the least restrictive noise level requirement. Second, we prove that the objective value and the sequence of iterates of GPM converge linearly to the optimal value and the unique global optimizer, respectively, under a less restrictive requirement compared with that in the literature. This answers an open question raised in a recent work Boumal.

A Simulation Optimization on the Hierarchical Health Care Delivery System Patient Flow Based on Multi-fidelity Models

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Abstract. The mismatching patient flow distribution in the health care system in urban China is a great social issue that attracts lots of public attention. In this research, we propose a simulation-based optimization method using the multi-fidelity optimization with ordinal transformation (OT) and optimal sampling (OS) (MO2TOS) algorithm to e-valuate the patient flow distribution, so as to continuously improve the hierarchical health care service system. The low-fidelity model applying the queueing network theory is constructed for the OT part of the MO2TOS, followed by a high-fidelity but time-consuming discrete event simulation model for the OS part. An empirical study on the patient flow data of the hierarchical health care delivery system in China is presented, where the proposed MO2TOS method is implemented to optimize the system profit by guiding the patient flow distribution. A comparison with other widely used simulation optimization methods sustains the efficacy of the MO2TOS with the evidence that acquiring effective information from the low-fidelity model indeed retrenches the computing budget used to explore the feasible domain.

Full Tensor Magnetic Gradient Data Inversion With Sparse Optimization

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Abstract. Traditional magnetic inversion is based on the total magnetic intensity (TMI) data and solving the corresponding mathematical physical model to obtain the related parameters. In recent years, with the development of the superconducting quantum interference device (SQUID), acquisition of the full tensor gradient magnetic data becomes available. In China, we have designed a low-temperature SQUID system, and performed the field work in 2016. Therefore, in this talk, we report our first results on the inversion of magnetic parameters using the measured full tensor magnetic gradient data. This is a very large scale problem. A Tikhonov regularization model with prior constraint is established. Gradient descent method with projections onto convex set is used to solve the minimization model. Experimental tests on synthetic data and field data are performed to show the advantage of the inversion with full tensor gradient magnetic data.

Stochastic First-Order Methods in Data Analysis and Reinforcement Learning

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Abstract. Stochastic first-order methods provide a basic algorithmic tool for online learning and data analysis In this talk, we survey several innovative applications including risk-averse optimization, online principal component analysis, and reinforcement learning. We will show that rate of convergence analysis of the stochastic optimization algorithms provide sample complexity analysis for these online learning applications.

Global Optimization With Orthogonality Constraints

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Abstract. Orthogonality constrained problems have wide applications in science and engineering. The nonconvex orthogonality constraints makes these problems hard to achieve their global optimizers. We aim at establishing an efficient algorithm for finding global minimizers under one or more orthogonality constraints. The main concept is based on noisy gradient flow constructed from SDE on the Stiefel manifold. The effectiveness and efficiency of the proposed algorithms are demonstrated on a variety of problems including homogeneous polynomial optimization, computation of stability number, iterative closest point (ICP) and 3D structure determination from Common Lines in Cryo-EM.

Boosting with Structural Sparsity – A Differential Inclusion Approach

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Abstract. Boosting, as gradient descent method, is arguably the 'best off-the-shelf methods in machine learning. Here a novel Boosting-type algorithm is proposed based on restricted gradient descent whose underlying dynamics are governed by differential inclusions. In particular, we present an iterative regularization path with structural sparsity where the parameter is sparse under some linear transforms, based on variable splitting and the Linearized Bregman Iteration. Hence it is called Split LBI. Despite its simplicity, Split LBI outperforms the popular generalized Lasso in both theory and experiments. A theory of path consistency is presented that equipped with a proper early stopping, Split LBI may achieve model selection consistency under a family of Irrepresentable Conditions which can be weaker than the necessary and sufficient condition for generalized Lasso. Furthermore, some l2-error bounds are also given at the minimax optimal rates. The utility and benefit of the algorithm are illustrated by applications on image reconstruction, partial order ranking, and Alzheimers disease detection via neuroimaging.

How to Implement ADMM to Large-scale Datasets?

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Abstract. The alternating direction method of multipliers (ADMM) is being popularly used for a wide range of applications including many in data science. To tackle very large-scale datasets of some representative statistical learning problems such as the distributed LASSO, it is neither possible nor necessary to solve the ADMM subproblems exactly or up to a high precision — inexact solutions in low-accuracy are cheaper while indeed better towards the convergence. We try to study this implementation issue mathematically. We are particularly interested in the case where the subproblems are very large-scale systems of linear equations and they are solved iteratively by benchmark numerical linear algebra solvers such as CG or SOR. We show how to rigorously ensure the convergence for this case. More specifically, we estimate precisely how many iterations of these numerical linear algebra solvers are needed to ensure the convergence. We thus make the implementation of ADMM embedded with benchmark numerical linear algebra solvers for the large-scale cases of some statistical learning problems fully automatic, with proved convergence.

Contributed Talks

An Optimized Framework for Crop Yield Estimation based on Decision Support System for Agrotechnology Transfer (DSSAT v4.6) Model

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Abstract. In this paper we discuss an integrated approach to developing a conceptualization framework for Decision Support System for Agrotechnology Transfer (DSSAT) Software Application, which is used in around 100 countries by agronomists to evaluate farming methods. DSSAT is modelled to enable crop yield estimation system in scientific and time precise manner focusing on precision agriculture. The in-season crop yield estimation and forecasting plays a crucial role for planners and policy makers in decision making regarding food production and early warning systems for taking adequate preventive measures. For any crop simulation model, data preparation is one of the main component to start the simulation. It requires a dedicated amount of time to convert this processed data into crop model format. To prepare data for DSSAT model which requires daily weather data, soil surface & profile information, detailed crop management as input. The experiment file modules have to be run for each location on an individual basis. We have developed a framework to optimize this process of data preparation involving many parameters. An Application program have been developed using MATLAB which prepares data for every location as a single process instead of running multiple times. Thereafter it runs the experimental simulation in Batch Mode simultaneously.

Keywords: DSSAT, Optimization, Simulation, Batch Mode, Precision Agriculture

Fast Decoding via Randomly Projected Sparse Retrieval

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Abstract. A binary word w of length k can be encoded as a word z of length n (with $k < n$) such that $z = Qw$ where Q is an $n \times k$ real matrix, which we assume to have rank k. After transmission on an analogue noisy channel the other party receives $\bar{z}$. We assume $\bar{z} = z + \bar{x}$, where $\bar{x}_j$ is uniformly distributed in $[-\delta, \delta]$ for some given $\delta > 0$ with some given (reasonably small) probability $\epsilon > 0$, and $\bar{x}_j = 0$ with probability $1 - \epsilon$.

The decoding of $\bar{z}$ into w is carried out as follows. We find a $k \times n$ matrix A orthogonal to Q (so $AQ = 0$), we compute $b = A\bar{z}$, then solve the ℓ_1 norm minimization "sparse retrieval" Linear Program (LP):

$$\min\{\|x\|_1 \mid Ax = b\}. \qquad (*)$$

We obtain an optimum x^* and use it to compute $y = \bar{z} - x^*$. Now we can recover the original binary message w as the integer part of $(Q^\top Q)^{-1}(Q^\top y)$.

I will talk about an application of random projections to this setting. With P.L. Poirion (Huawei, France) and Vu Khac Ky (CUHK), we recently derived some new results about approximate feasibility and optimality invariance when replacing linear constraints $Ax = b$ by their randomly projected version $TAx = Tb$ (where T is a random projection matrix such as those used in the Johnson-Lindenstrauss lemma). It turns out $(*)$ can be replaced with its much smaller randomly projected version with no precision loss up to considerable sizes.

An Alternating Minimization Method for Robust Principal Component Analysis

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Abstract. We focus on solving robust principal component analysis (RPCA) arising from various applications such as information theory, statistics, engineering, and etc. We adopt a model to minimize the sum of observation error and sparsity measurement subject to the rank constraint. To solve this problem, we propose a two-step alternating minimization method. In one step, a symmetric low rank product minimization, which essentially is partial singular value decomposition, can be efficiently solved to moderate accuracy. Meanwhile the second step has closed-form solution. The new proposed approach is almost parameter-free, and its global convergence to a strict local minimizer can be derived under almost no assumption. We compare the proposed approach with some existing solvers and the numerical experiments demonstrate the outstanding performance of our new approach in solving a bunch of synthetic and real RPCA test problems. Especially, we discover the great potential of the proposed approach in solving problem with large size to moderate accuracy.

Workshop 3

Modeling and Computation for Flow and Transport in Porous Media

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Invited Talks

MRST-co2lab: Rapid Prototyping of Engineering Solutions for Problems Related to Large-scale Aquifer-wide CO_2 Storage

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Abstract. Greenhouse gases, in particular carbon dioxide (CO_2), have been blamed for the rise in earth surface temperatures leading to climate change. To prevent further accumulation of CO_2 in the atmosphere, this gas can be captured at industrial or power-generating point sources, transported in its supercritical state, and injected into the subsurface for long-term storage. This concept, known as carbon capture and storage (CCS), was first put into practice two decades ago by the Sleipner project, located in the Norwegian North Sea. Since its commencement, around 16 Mt of CO_2 have been injected in a layered formation under the seabed, and seismic imaging has shown the storage to be secure. However, given that our global annual CO_2 emissions was around 36 Gt in 2015 [1], there is a consensus in the CCS community that more and larger storage projects are needed to handle our projected global emissions.

An important question to answer before starting a CCS project is how much CO_2 may be adequately stored in a saline aquifer. One cannot simply look at the pore volume of a formation to find a practical capacity estimate. Rather, realistic capacity estimates must consider the flow dynamics involved during the injection and post-injection periods. As such, mathematical models and computational methods play an important role in assessing storage capacity. Numerical simulations can model CO_2 migration throughout the geomodel and the various trapping mechanisms that occur as a result. Simulations help provide insight into which engineering solutions are most appropriate to enhance the storage capacity of a saline aquifer, such as the use of brine production for pressure management, brine re-injection or cycling. Practical storage capacities depend on many variables (e.g., number of wells, well rates, length of injection, well placement, model parameters, etc.), and to assess a variety of different injection strategies or model parameters, one needs a fluid flow model that is computationally cheap yet accurate with

respect to the scale of the problem. That is, determining the ultimate state of the injected CO_2 requires tracking its migration for thousands for years, during which it may spread over hundreds of kilometers. The need to consider such large temporal and spatial scales means reduced-order models become an attractive alternative to full 3D modeling.

A module of MATLAB Reservoir Simulation Toolbox (MRST) [2], known as MRST-co2lab [3], is comprised of tools to help answer engineering and operational related questions regarding large-scale CO_2 storage. These tools include spill-point analysis, vertical equilibrium (VE) modeling, migration forecasting, and mathematical optimization. By using these features, the user is able to estimate an aquifer's structural trapping potential, rapidly simulate CO_2 injection and plume migration, forecast the ultimate fate of the migrating plume without needing to simulate many years into the future, optimize well injection and/or production rates, and perform model calibration. In this talk, examples will demonstrate how MRST-co2lab may be used to obtain optimized well rates given leakage and pressure constraints, implement CO_2 injection with brine production for pressure management, compare the impact of well placement on the optimized storage capacity, obtain the lowest risk injection strategy that factors in parameter uncertainty, and perform model calibration of the Sleipner benchmark problem.

In summary, this talk demonstrates by example the tools that are part of MRST-co2lab, which can be used for rapid prototyping of possible engineering solutions to enhance the storage capacity of a large-scale saline aquifer. The reduced-order flow model is faster than traditional 3D models, yet still represents the physics of two-phase flow with gravity segregation in porous media. Rapid simulations mean it is possible to solve problems that require a large number of iterations, e.g., well optimization, uncertainty quantification, or model calibration, within a reasonable amount of time.

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Polymer Flooding Simulation With MRST

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Abstract. In the presentation, we describe a fully implicit simulator for polymer injection implemented in the free, open-source MATLAB Reservoir Simulation Toolbox (MRST). Polymer injection is one of the widely used enhanced oil recovery (EOR) techniques and complicated physical process is involved, which makes accurate and efficient simulation very challenging. The proposed work is intended for providing a powerful and flexible tool to investigate the polymer injection process in realistic reservoir scenarios.

Within the model, the polymer component is assumed to be only transported in the water phase and adsorbed in the rock. The hydrocarbon phases are not influenced by the polymer and they are described with the standard, three-phase, black-oil equations. The effects of the polymer are simulated based on the Todd–Longstaff mixing model, accounting for adsorption, inaccessible pore space, and permeability reduction effects. Shear-thinning/thickening effects based on shear rate are also included by the means of a separate inner-Newton iteration process within the global nonlinear iteration. The implementation is based on the automatic differentiation framework in MRST (MRST-AD), and an iterative linear solver with a constrained pressure residual (CPR) preconditioner is used to solve the resulting linear systems efficiently.

Certain implementation details are discussed to show how convenient it is to use the existing functionality in MRST to develop an accurate and efficient polymer flooding simulator for real fields. With its modular design, vectorized implementation, support for stratigraphic and general unstructured grids, and automatic differentiation framework, MRST is a very powerful prototyping and experimentation platform for development of new reservoir simulators.

First, verification against a commercial simulator is performed and good agreement is achieved. Then, we apply the new simulator to a few realistic reservoir models to investigate the effect of polymer injection and computational efficiency is demonstrated. Finally, we combine existing optimization functionality in MRST with the new polymer simulator to optimize polymer flooding for two different reservoir models. We argue that the presented software framework can be used as an efficient prototyping tool to evaluate new models for polymer-water-flooding processes in real reservoir fields.

Finally, as a demonstration that the simulation framework is a good platform for testing new computational methods, we briefly introduces an efficient multiscale restricted-smoothed basis (MsRSB) method for polymer flooding based on a sequentially implicit formulation instead of the fully implicit formulation, which appears to be more robust and stable than the fully-implicit approach. With the method, the computational time is dominated by the pressure solution, and 5-8 times speedup is observed when replacing

the fine-scale pressure solution by the iterative multiscale method.

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Fractal Characterization of Capillary Driven Flow in Porous Media

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Abstract. Spontaneous imbibition of wetting liquid in porous media is a ubiquitous natural phenomenon, in which the wetting liquid imbibed into pores derived by capillary pressure. This capillary-driven flow in porous media has received more and more attentions in many fields, such as soil physics, petroleum engineering, polymeric composites and biological science. The capillary pressure is related to the pore size and structure. However, natural porous media usually have extremely complicated microstructure, which make the imbibition process and flow mechanism are not well characterized and understood. Fortunately, Fractal geometry has been verified to be useful tool to describe porous media and its transport properties. By including fractal theory, an analytical spontaneous imbibition model is developed based on Hagen-Poiseuille equation and the assumption of capillary tube model. The imbibition dynamics is analytically related to the microstructure of porous media. Available imbibition experimental data are used to test the developed fractal model. In last, Future directions and recommends are also discussed.

Reduced Models for Flow in Fractured Porous Media: Adaptation and Two-scale Model

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Abstract. In this talk we will introduce an a posteriori error estimator for the Raviart-Thomas mixed finite element method for single-phase Darcy flow in a two-dimensional fractured porous media. The discrete fracture model (DFM) is applied to model the fractures by one-dimensional fractures in a two-dimensional domain. We derive a robust residual-based a posteriori error estimator for the problem with non-intersecting fractures. The reliability and efficiency of the a posteriori error estimator are established for the error measured in an energy norm. We will also introduce a two-scale reduced model for simulating the Darcy flow in the two-dimensional porous media with conductive fractures. We apply the approach motivated by the embedded fracture model (EFM) to simulate the flow on the coarse scale, and the effect of fractures on each coarse scale grid cell intersecting with fractures is represented by the DFM on the fine scale. Several numerical results will be shown to demonstrate the efficiency of the adaptive algorithm and the proposed two-scale model.

Adaptive Mixed-hybrid and Penalty Discontinuous Galerkin Method for Two-phase Flow in Heterogeneous Media

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Abstract. In this paper, we present a hybrid method, which consists of a mixed-hybrid finite element (MHFE) method with BDM_1 space and a penalty discontinuous Galerkin (PDG) method, for the approximation of a fractional flow formulation of a two-phase flow problem in heterogeneous media with discontinuous capillary pressure. The fractional flow formulation is comprised of a wetting phase pressure equation and a wetting phase saturation equation which are coupled through a total velocity and the saturation affected coefficients. For the wetting phase pressure equation, the continuous MHFE method with BDM_1 can be utilized due to a fundamental property that the wetting phase is continuous. While it can reduce the computational cost by using less degrees of freedom and avoiding the post-processing of velocity reconstruction, this method can also keep several good properties of the discontinuous Galerkin (DG) method, which are important to the fractional flow formulation, such as the local mass balance, continuous normal flux and capability of handling the discontinuous capillary pressure. For the wetting phase saturation equation, the PDG method is utilized due to its capability of handling the discontinuous jump of the wetting phase saturation. Furthermore, an adaptive algorithm for the hybrid method together with the centroidal Voronoi Delaunay triangulation (CVDT) technique is proposed. Four two dimensional numerical examples are presented to illustrate the features of proposed numerical method, such as the optimal convergence order, the accurate and efficient velocity approximation, and the applicability to the simulation of water flooding in oil field and the oil-trapping or barrier effect phenomena.

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Mixed Generalized Multiscale Finite Element Methods for Flows in Heterogeneous Media

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Abstract. In this talk, we present a mixed Generalized Multiscale Finite Element Method (GMsFEM) for solving flow in heterogeneous media. Our approach constructs multiscale basis functions following a GMsFEM framework and couples these basis functions using a mixed finite element method, which allows us to obtain a mass conservative velocity field. To construct multiscale basis functions for each coarse edge, we design a snapshot space that consists of fine-scale velocity fields supported in a union of two coarse regions that share the common interface. The snapshot vectors have zero Neumann boundary conditions on the outer boundaries and we prescribe their values on the common interface. We describe several spectral decompositions in the snapshot space motivated by the analysis. We present numerical results for two-phase flow and transport, without updating basis functions in time. Our numerical results show that one can achieve good accuracy with a few basis functions per coarse edge if one selects appropriate offline spaces.
This work is partially supported by the Hong Kong RGC General Research Fund (Project: 14317516).

Multiscale Model Reduction for Flow and Transport in Porous Media

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Abstract. In this talk, I will discuss multiscale model reduction techniques for problems in heterogeneous media. I will discuss homogenization-based multiscale methods and their relations to multiscale finite element methods. I will describe a general multiscale framework for constructing local (space-time) reduced order models for problems with multiple scales and high contrast and their relation to multi-continuum approaches. Some issues related to the construction of multiscale basis functions, main ingredients of the method, and a number of applications will be described. A generalization of these approaches to stochastic problems will also be discussed.

A Data-driven Multiscale Finite Volume Method for Uncertainty Quantification in Multiphase Flow Problems

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Abstract. Several multiscale methods account for sub-grid scale features using coarse scale basis functions. For example, in the Multiscale Finite Volume method the coarse scale basis functions are obtained by solving a set of local problems over dual-grid cells. We introduce a data-driven approach for the estimation of these coarse scale basis functions. Specifically, we employ a neural network predictor fitted using a set of solution samples from which it learns to generate subsequent basis functions at a lower computational cost than solving the local problems. The computational advantage of this approach is realized for uncertainty quantification tasks where a large number of realizations has to be evaluated. We attribute the ability to learn these basis functions to the modularity of the local problems and the redundancy of the permeability patches between samples. The proposed method is evaluated on several multiphase flow problems yielding very promising results.

An Efficient Multiscale Method for Modeling Flow in Fractured Vuggy Reservoirs

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Abstract. Fractures and vugs play a significant effect on the macro-scale flow, thus should be described exactly. Accurate modeling of flow in fractured media is usually done by discrete fracture model (DFM) and discrete fractured vuggy model (DFVN). Both models can provide detailed representation of flow characteristic. However, considering the computational efficiency and accuracy, traditional numerical methods are not suitable for DFM and DFVN. In order to extend the DFM and DFVN to field-scale applications, in our work, multiscale methods, including multiscale finite element method (MsFEM) and multiscale mixed finite element method (MsMFEM), are proposed for detailed modeling of fluid flow in fractured vuggy reservoirs. In multiscale methods, the governing equations are solved on coarse grid. The interaction between the fractures, vugs and the matrix is captured through multiscale basis functions calculated numerically by solving DFM or DFVN on the local fine grid. Through multiscale basis functions, multiscale methods can not only reach a high efficiency as upscaling technology, but also finally generate a more accurate and conservative velocity field on the full fine-scale grid. In our approach, oversampling technique is applied to get more accurate small-scale details. Triangular fine-scale grid is applied, making it possible to consider fractures in arbitrary directions. Comparisons of the multiscale solutions with the full fine-scale solutions indicate that the later one can be replaced by the former one. The results showed that the multiscale technology is a promising numerical method for multiscale flows in high-resolution fractured media.

Modeling of Thermal Transport in Metal Foam and Its Application

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Abstract. Due to the large specific area, metal foam can greatly extend the convective heat transfer area. Meantime, the complex structure can induce a strong flow-mixing capability and decrease the thermal boundary layer thickness. Thus, metal foams have a promising future in many thermal engineering applications, such as electronic cooling, thermal energy absorbers, solar thermal energy utilization, compact heat changers, and so on. Among these, efficient and compact heat exchangers and heat sinks are favorable in many engineering applications, which can motivate corresponding research interest. In this work, analysis of thermal modeling of fluid flow and heat transfer in metal foam will investigated in detail and its application to electronic cooling will also be numerically studied. Firstly, the mechanisms of fluid flow and heat transfer in metal foam are systematically analyzed by adopting different models including Darcy model, Brinkman model and Frochheimer model for fluid flow and LTE model, LTNE model for heat transfer, and the corresponding revised model is also proposed to improve the predicting accuracy. In addition, in light of the great enhancement of fluid thermal conductivity of nano particles, the combination of meat foam and nano-fluid can be treated as a dual heat transfer enhancement method by both improving the solid and fluid thermal conductivity, and its thermal performance is investigated in detail. Finally, the application of metal foam as a novel heat sink to electronic cooling is numerically studied and compared with the conventional heat sink.

Multi-scale Diffuse Interface Modeling Of Multi-component Two-phase Flow With Partial Miscibility

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Abstract. We introduce a diffuse interface model to simulate multi-component two-phase flow with partial miscibility based on a realistic equation of state (e.g.Peng-Robinson equation of state). Because of partial miscibility, thermodynamic relations are used to model not only interfacial properties but also bulk properties, including density, composition, pressure, and realistic viscosity. As far as we know, this effort is the first time to use diffuse interface modeling based on equation of state for modeling of multi-component two-phase flow with partial miscibility. In numerical simulation, the key issue is to resolve the high contrast of scales from the microscopic interface composition to macroscale bulk fluid motion since the interface has a nanoscale thickness only. To efficiently solve this challenging problem, we develop a multi-scale simulation method. At the microscopic scale, we deduce a reduced interfacial equation under reasonable assumptions, and then we propose a formulation of capillary pressure, which is consistent with macroscale flow equations. Moreover, we show that Young-Laplace equation is an approximation of this capillarity formulation, and this formulation is also consistent with the concept of Tolman length, which is a correction of Young-Laplace equation. At the macroscopical scale, the interfaces are treated as discontinuous surfaces separating two phases of fluids. Our approach differs from conventional sharp-interface two-phase flow model in that we use the capillary pressure directly instead of a combination of surface tension and Young-Laplace equation because capillarity can be calculated from our proposed capillarity formulation. A compatible condition is also derived for the pressure in flow equations. Furthermore, based on the proposed capillarity formulation, we design an efficient numerical method for directly computing the capillary pressure between two fluids composed of multiple components. Finally, numerical tests are carried out to verify the effectiveness of the proposed multi-scale method.

Mathematical Modeling of Water Transport in Thermochemical Materials

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Abstract. Thermal energy storage is a key ingredient in efficient utilization of the solar energy. While the solar heat is abundant in summer, when the demand is low, the need for heat is high in winter, when the amount of available solar heat is limited. To resolve the mismatch between the availability of the solar heat in summer and the demand for the heat in winter, seasonal heat storage systems can be considered. One of the available options is to store energy in the form of a reversible chemical reaction in a thermochemical material. A typical reaction of this kind reads as:

$$\mathrm{H_2O} + \text{zeolite} \rightleftharpoons \text{zeolite}.\mathrm{H_2O} + \text{heat}.$$

The solar energy can be stored in summer by heating the zeolite and separating the produced water vapor. The dried zeolite is stored for later use. In winter the stored energy can be released by bringing the dried zeolite into contact with the wet air. The advantage over other heat storage technologies is that there is no need for thermal insulation of the thermochemical material as the energy is stored in the form of the chemical bonds and not the heat. Zeolite is a stable nanoporous material with relatively high storage capacity. To increase our understanding of the processes occuring in the zeolites during the seasonal heat storage, we have developed a matematical model describing the transport and adsorption of water vapor in a bead of the zeolite. We are going to present a formulation of model equations, describe numerical methods used for the discretization of the problem, and present first numerical results simulating water transport in a bead of zeolite.

Numerical Methods for Coupled Stokes and Darcy Flows With Transfer

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Abstract. In this article, we present some stabilized mixed finite element method for solving the coupled Stokes and Darcy flow equations with a solute transfer. The mathematical model includes the velocity and pressure equations and concentration equation where the viscosity depends on the concentration. We propose a mixed weak formulation and use the nonconforming piecewise Crouzeix-Raviart finite element, piecewise constant and conforming piecewise linear finite element to approximate velocity, pressure and concentration respectively. The existence, uniqueness of the approximate solution are obtained, and optimal order a priori error estimates are derived. No assumption on the boundness of infinitive norms of approximate velocity or concentration or the restriction about the time-step and spatial meshsize is needed. Numerical examples are presented to verify the theoretical results.

Fast Numerical Methods for Peridynamic Models

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Abstract. The peridynamic (PD) theory is a reformulation of the classical theory of continuum solid mechanics, and yields nonlocal mathematical formulations. Constitutive models in the PD theory depend on finite deformation vectors, instead of deformation gradients in classical constitutive models. Consequently, PD models are particularly suitable for the representation of discontinuities in displacement fields and the description of cracks and their evolution in materials. However, because of the nonlocality of the PD operators, the numerical methods for PD models yield dense stiffness matrices. Asymptotically, these methods have $O(N^2)$ memory requirement and the widely used direct solvers for PD models have $O(N^3)$ computational complexity to invert the coefficient matrices. On the other hand, the matrix-vector multiplication by the stiffness matrices have $O(N^2)$ computational complexity. Furthermore, the evaluation and assembly of the stiffness matrices require the evaluation of $O(N^2)$ entries, which can be very expensive and often constitute a very large portion of simulation times! This renders the numerical simulation of the PD models computationally very expensive, especially for problems in multiple space dimensions. In this talk we present the development of fast numerical methods for different PD models.

Acceleration of Single-Phase Flow Simulation in Porous Media Using Proper Orthogonal Decomposition

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Abstract. Numerical simulation of fluid flow in porous media is usually time-consuming. It cannot satisfy the requirement of fast prediction in engineering. Proper orthogonal decomposition (POD) method has been widely applied to flow and heat transfer problems in engineering. It can largely reduce the degrees of freedom of a problem so that the calculation of large partial differential equation system for the problem is transformed to the calculation of small linear equation system in low-dimensional space for the same problem. Computational time is therefore greatly reduced via this transformation. The linear equation system is called reduced-order model. In the reduced-order model, variables are the linear combination of POD modes and corresponding coefficients. The POD modes can be calculated from sample data using singular value decomposition. The coefficients are usually obtained by solving the reduced-order model established via Galerkin projection. The use of Galerkin projection is because it can be used for both steady-state and unsteady-state problems. Typical single-phase flow in porous media includes incompressible single-phase flow for liquid and compressible single-phase flow for gas. Their governing equations are steady-state and unsteady-state in Darcys law. Thus, POD method combined with Galerkin projection is utilized in this study to establish reduced-order models for incompressible single-phase flow and gas flow in porous media. The utilization is expected to accelerate simulations of single-phase flow in porous media.

For incompressible single-phase flow in porous media, the POD reduced-order model shows high precision and large acceleration. Relative deviations of POD results with the results obtained from the original partial differential equation system are as low as $1.0\times 10^{-4}\%\sim 2.3\times 10^{-1}\%$.POD speeds up the computation as high as 880 98454 times. For compressible single-phase flow in porous media, reconstruction results of POD are also accurate (relative deviation 1.14%) but acceleration of computational speed is strictly restricted due to the majority of the computation on equation of state (EOS). Compressibility causes the reduction of equation system failed. The method to accelerate the computation of EOS should be developed in future.

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3D Parallel Wormhole Simulation with the Darcy-Brinkman-Forchheimer Framework

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Abstract. Matrix acidization is an important technique in petroleum engineering to increase the oil recovery rate. Different from the numerical simulation of the traditional Darcy framework, the Darcy-Brinkman-Forchheimer (DBF) framework should simulate matrix acidization with more reasonable results. This work extends the existing 2D model of the DBF framework to the 3D case in order to satisfy the real industry need. However, the 3D realization is not a simple extension of the 2D model, and it includes a lot of complicated details. This work carefully describes the realization details of discretization and parallelization of the 3D model. The 3D code is also verified by the benchmark of the 3D shear-driven cavity flows. Then, the 3D dissolution experiment is carried out and a comparison with the 2D experiment is presented. At last, 3D parallel performance is evaluated.

Active-set Reduced-space Methods With Nonlinear Elimination for Two-phase Flow Problems in Porous Media

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Abstract. Most existing methods for solving two-phase flow problems in porous media do not take the physically feasible saturation fractions between 0 and 1 into account. In the talk, we reformulate the two-phase model as a variational inequality that naturally ensures the physical feasibility of the saturation variable. Under the frame of the fully implicit method, the variational inequality is then solved by an active-set reduced-space method with a nonlinear elimination preconditioner to remove the high nonlinear components that often causes the failure of the nonlinear iteration for convergence. To validate the effectiveness of the proposed method, we compare it with the classical IMplicit Pressure-Explicit Saturation (IMPES) method for two-phase flow problems with strong heterogeneity. The numerical results show that our nonlinear solver overcomes the often severe limits on the time step associated with existing methods, results in superior convergence performance, and achieves reduction in the total computing time by more than one order of magnitude.

Localized Solvers For Implicit Time-Stepping

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Abstract. In the solution of implicit reservoir simulation timesteps, the Newton iteration updates are often very sparse; this sparsity can be as high as 95% and can vary dramatically from one iteration to the next. We develop, implement and demonstrate a mathematically sound adaptive framework to predict this sparsity pattern before the system is solved. The development first mathematically relates the Newton update in functional space to that of the discrete system. Next, the Newton update formula in functional space is homogenized and solved in such a way that it results in conservative estimates of the numerical Newton update. The cost of evaluating the estimates is linear in the number of nonzero components. The estimates are used to label the components of the solution vector that will be nonzero, and the corresponding submatrix is solved. The computed result is guaranteed to be identical to the one obtained by solving the entire system.

When applied to various simulations of three-phase flow recovery processes in the SPE 10 geological model, the observed reduction in computational effort ranges between four to tenfold depending on the level of total compressibility in the system, the time step size and on the degree of complexity in the underlying physics. We show the extensions to the case of flow and multicomponent transport where the reduction in computation effort ranges between four to tenfold. The improvement in computational speed scales strongly with the number of transport components, and to a lesser degree with problem size. The results of the localized and full simulations are identical, as is the nonlinear convergence behavior.

A Class of Conforming Multiscale Finite Element Method for Elliptic Problems With Multiscale Coefficients

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Abstract. We develop a conforming Multiscale finite element method for second order elliptic equations with rough or highly oscillatory coefficients. Our method is based on an iterative approach that enables us to obtain more accurate boundary conditions for the local cell problems. Several numerical examples are provided to show the accuracy and convergence of the proposed method.

Efficient Two-phase Compositional Fractured Reservoir Simulation in 2D and 3D Unstructured Gridding

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Abstract. Efficient numerical modeling is advanced for simulation of compositional two-phase flow in fractured media with species transfer between the phases on unstructured gridding in 2D and in 3D. The robustness of the model is demonstrated by a set of numerical examples.

Our model is based on the implicit and explicit time discretization. Implicit time discretization overcomes the CFL (Courant-Freidricks-Levy) condition in small fracture grid cells. The implicit discretization of the transport equation is based on the calculation of the derivative of the molar concentration of component i in phase α ($c_{a,i}$) with respect to the total molar concentration (c_i). The explicit discretization relates to the discontinuous Galerkin methods in the mass balance equations in the rock matrix. The mixed finite element is used to evaluate the fluxes both in the fractures and the rock matrix. Rectangular and triangular elements are used in 2D, and hexahedra, prisms and tetrahedral elements in 3D.

Various numerical examples are presented to demonstrate the efficiency and accuracy of the model. We show that our model is stable on different types of grids in 2D rectangular and triangular elements, and in 3D hexahedron and prism elements. Even in complex geometries discretized by unstructured tetrahedral elements, no convergence problems are observed. The model has very low mesh dependency in problems with different complexities. The robustness of our formulation is due to small variation of the nonlinear coefficients of transport equation in the Newton method discretization and the accuracy of the higher-order methods.

The novelty of our proposed algorithm is the ability to simulate fractured reservoirs in two-phase compositional flow in 2D and 3D complicated geometries using unstructured gridding in a very efficient and accurate way. Unlike commercial simulators which have serious convergence issues, we always observe convergence. The proposed algorithm is not only much more accurate than commercial simulators but also due to faster convergence is at least one order of magnitude faster than all the commercial simulators.

Contributed Talks

A Lattice Boltzmann Model for Multiphase Flows With Large Density Ratio on Rough Surface

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Abstract. In this paper, we develop an efficient lattice Boltzmann model for the two-phase flows with large density ratio on rough surface. The Navier-Stokes and Cahn-Hilliard equations are recovered from the lattice Boltzmann model. In order to describe the behavior of the contact line motion on the boundary, we incorporate the generalized Navier boundary condition by the nonequilibrium extrapolation method. The proposed method is easy to implement and retains the advantage of the standard lattice Boltzmann method. Numerical tests are carried out to verify the proposed method. Our numerical results show that the present approach is able to model two-phase flows with large density ratio on rough surface.

Stokes-dual-porosity Model and Finite Element Method for Coupling Dual-porosity Flow and Free Flow

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Abstract. We propose and numerically solve a new model considering confined flow in dual-porosity media coupled with free flow in embedded macro-fractures and conduits. Such situation arises, for example, for fluid flows in hydraulic fractured tight/shale oil/gas reservoirs. The flow in dual-porosity media, which consists of both matrix and micro-fractures, is described by a dual-porosity model. And the flow in the macro-fractures and conduits is governed by the Stokes equation. Then the two models are coupled through four physically valid interface conditions on the interface between dual-porosity media and macro-fractures/conduits, which play a key role in a physically faithful simulation with high accuracy. All the four interface conditions are constructed based on fundamental properties of the traditional dual-porosity model and the well-known Stokes-Darcy model. The weak formulation is derived for the proposed model and the well-posedness of the model is analyzed. A finite element semi-discretization in space is presented based on the weak formulation and four different schemes are then utilized for the full discretization. Some numerical experiments are presented to validate the proposed model and demonstrate the features of both the model and numerical method, such as the optimal convergence rate of the numerical solution, the detail flow characteristics around macro-fractures and conduits, and the applicability to the real world problems.

JRC Workshop

Uncertainty, Risk and Control

Invited Talks

Stochastic Global Maximum Principle for Optimization With Recursive Utilities

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Abstract. We study the recursive stochastic optimal control problems. The control domain does not need to be convex, and the generator of the backward stochastic differential equation can contain z. We obtain the variational equations for backward stochastic differential equations, and then obtain the maximum principle which solves completely Peng's open problem.

Representation of Asymptotic Values for Nonexpansive Stochastic Control Systems

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Abstract. We will study the limit behavior of the Abel mean $\lambda V_\lambda(x)$ which is introduced by BSDE as the discount factor λ tends to zero. We show that the family of functions $\{\lambda V_\lambda(x)\}_\lambda$ is equicontinuous and equibounded under appropriate conditions. And we characterize the limit value of $\lambda V_\lambda(x)$. It's based on a joint work with Nana Zhao (Shandong University, Weihai).

Some Properties of G-Itô Process and Their Applications

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Abstract. In this talk, we use partial differential equation (PDE) tech-niques to study the lower capacity when the G-Itô process stays in the ball. We also prove the strict comparison theorem and strict sub-additivity in G-expectation framework.
This talk is based on the joint works with Yiqing LIN and Falei WANG.

Quadratic Backward Stochastic Differential Equations Driven By G-brownian Motion: Discrete Solutions And Approximation

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Abstract. In this paper, we consider backward stochastic differential equations driven by G-Brownian motion (GBSDEs) under quadratic assumptions on coefficients. We prove the existence and uniqueness of solution for such equations. On the one hand, a priori estimates are obtained by applying the Girsanov type theorem in the G-framework, from which we deduce the uniqueness. On the other hand, to prove the existence of solutions, we ?rst construct solutions for discrete GBSDEs by solving corresponding fully nonlinear PDEs, and then approximate solutions for general quadratic GBSDEs in Banach spaces.

Joint work with YING HU and ABDOULAYE SOUMANA HIMA.

Linear-Quadratic-Gaussian Mixed Games with Input Constraint Involving Major Agent and Heterogeneous Minor Agents

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Abstract. We consider a class of linear-quadratic-Gaussian mean-field games with a major agent and considerable heterogeneous minor agents with mean-field interactions. The individual admissible controls are constrained in closed convex subsets of the full space. The decentralized strategies for individual agents and the consistency condition system are represented in an unified manner via a class of mean-field forward-backward stochastic differential equations involving projection operators. The wellposedness of consistency condition system is established and the related ?-Nash equilibrium property is also verified. This is a joint work with Prof. Ying Hu and Prof. Jianhui Huang.

Connection between MP and DPP for Stochastic Recursive Optimal Control Problems

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Abstract. This talk deals with a stochastic recursive optimal control problem, where the diffusion coefficient depends on the control variable and the control domain is not necessarily convex. After some review of classical results in the literatures, we focus on the connection between the general maximum principle and the dynamic programming principle for such control problem without the assumption that the value is smooth enough, the set inclusions among the sub- and super-jets of the value function and the first-order and second-order adjoint processes as well as the generalized Hamiltonian function are established. Moreover, by comparing these results with the classical ones in Yong and Zhou [Stochastic Controls: Hamiltonian Systems and HJB Equations, Springer-Verlag, New York, 1999], it is natural to obtain the first- and second-order adjoint equations of Hu [Direct method on stochastic maximum principle for optimization with recursive utilities, arXiv:1507.03567v1 [math.OC], 13 Jul. 2015].
This talk is based on a joint work with Prof. Tianyang Nie and Prof. Zhen Wu.

Backward Doubly Stochastic Volterra Integral Equations and Applications

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Abstract. In this talk, we introduce and study a class of integral equations so-called backward doubly stochastic Volterra integral equations (BDSVIEs in short). There are two independent Brownian motions W(t) and B(t) in the BDSVIE, where the dW-integral is a forward Itos integral and the dB-integral is a backward Ito's integral. In order to obtain the existence and uniqueness theorem of BDSVIEs, the notion of adapted martingale solution (M-solution in short) is introduced. Well-posedness of BDSVIEs in the sense of M-solutions is established. A comparison theorem for BDSVIEs is proved, and by virtue of the comparison theorem, we derive the existence of solutions for BDSVIEs with continuous coefficients. A duality principle between linear (forward) doubly stochastic Volterra integral equation (FDSVIE, for short) and BDSVIE is obtained. A Pontryagin type maximum principle is also established for an optimal control problem of FDSVIEs.

Stein Type Characterization for G-normal Distributions

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Abstract. In this talk, we provide a Stein type characterization for G-normal distributions.
A joint work with Mingshang Hu and Shige Peng.

Non-zero Sum Stochastic Differential Game of Bsde With Partial Information

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Abstract. In this talk, we introduce a non-zero sum stochastic differential game driven by BSDE, where the information available to the players is allowed to be asymmetric. A maximum principle as well as a verification theorem for Nash equilibrium point is derived. Some concrete examples including a financial one are solved explicitly.

This talk is based on the recent works Wang and Yu (Automatica 2012), Wang, Xiao and Xiong (arXiv 2017).

A Novel Multivariate Volatility Modeling for Risk Management

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Abstract. Volatility modeling is an influential area in financial econometrics and its central requirement is to forecast volatility accurately. The approaches of model averaging estimation are commonly used to achieve a satisfactory forecasting reliability. To reduce repetitive but unnecessary computational burden of these approaches, this talk introduces a fuzzy-method-involving multivariate volatility model (abbreviated as FMVM). It classifies the individual models into smaller scale clusters and selects the most representative model in each cluster. Numerical results show that it can obtain relatively lower forecasting errors with less model complexity.

Backward Stochastic Differential Equations Coupled With Two-time-scale Markov Chains and Applications in Optimal Switching Problem

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Abstract. This talk is concerned with backward stochastic differential equations (BSDEs) coupled by a finite-state Markov chains which has a two-time scale structure i.e. the states of the Markov chain can be divided into a number of groups so that the chain jumps rapidly within a group and slowly between the groups. In this talk, we give a convergence result as the fast jump rate goes to infinity, which can be used to reduce the complexity of the original problem. This method is also referred to as singular perturbation. The first result is the weak convergence of the BSDEs with two-time-scale BSDEs. It is proved that the solution of the original BSDE system converges weakly under the Meyer-Zheng topology. The limit process is a solution of aggregated BSDEs. The results are applied to a set of partial differential equations and used to validate their convergence to the corresponding limit system. And then we focus on the optimal switching problem for regime-switching model with two-time-scale Markov chains. Under the two-time-scale structure, we prove the convergence of the value functions (variational inequalities) and obtain the optimal switching strategy by virtue of the oblique reflected BSDEs with Markov chains. Numerical examples are given for the problem to demonstrate the approximation results.

Ioint work with Ran Tao and Qing Zhang.

The Stochastic Optimal Control Problem Under State Constraints

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Abstract. In this study, we consider an optimal control problem driven by a stochastic differential system with state constraints. Here, the state constraints mean the constraints on the path of state. In order to show the maximum principle for the optimal control problem under state constraints, we investigate a new near optimal control problem and establish the stochastic maximum principle for the new optimal control problem under multi-time state constraints. In the end, we give a production planning example to illustrate the main results of this study.

An Optimal Feedback Control-Strategy Pair of Zero-Sum Linear-Quadratic Stochastic Differential Game: the Riccati Equation Approach

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Abstract. In this paper, we study a two-person zero-sum linear-quadratic stochastic differential game problem. From a new viewpoint, we construct an optimal feedback control-strategy pair for the game in a closed-loop form based on the solution of a Riccati equation. A key part of our analysis involves proving the global solvability of this Riccati equation, which is interesting in its own right. Moreover, we demonstrate an inde?nite phenomenon arising from the linear-quadratic game.

Robust Portfolio Optimization Problems based on Laplace Distribution

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Abstract. In this paper, we consider a series robust portfolio optimization problems. For low-dimensional portfolio selection, instead of using multivariate Gaussian distribution of the traditional Markowitz's model, we introduced multivariate asymmetric Laplace distribution to the model, which alleviates the turbulence of financial market significantly, and stable efficient frontiers are obtained. For high dimensional portfolio selection problem, we applied LAD-lasso to the Mean-Absolute Deviation portfolio optimization, the special connection between LAD regression and univariate Laplace distribution improved the robustness of portfolio selection, and a steepest descent algorithm is proposed to speed up the calculation process.

Numerical Schemes for Stochastic Optimal Control Via FBSDEs

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Abstract. In this talk, based on the theories of optimization, stochastic optimal control and forward backward differential equations (FBSDEs), we will introduce some high accurate numerical schemes for solving stochastic optima control. In these schemes, the simplest Euler scheme is used to numerically solve the solutions of the forward stochastic differential equations, and multistep schemes is used to solve the backward stochastic differential equation (BSDE) with high convergence rate. Some stochastic optimal control models, coming from finance and economy, are solved by the schemes. Our numerical results show that our schemes are stable, accurate, and effective for solving stochastic optimal control problems.