An improved extra-gradient method for minimizing a sum of p-norms – a variational inequality approach

Xiaoming Yuan[1], Li Zhou[2]

[1] Department of Mathematics, City University of Hong Kong, Hong Kong
[2] Department of Mathematics, Nanjing University, Nanjing, China

This paper presents a variational inequality (VI) approach to the problem of minimizing a sum of p-norms. First the original problem is reformulated as an equivalent linear VI. Based on the special separate structure of the equivalent linear VI, an improved extra-gradient method is presented. At each iteration, the new method consists of a prediction and correction. The prediction step inherits the spirit of the classical Gauss-Seidel method for solving a system of nonlinear equations. Then the classical proximal point method is used to produce the corrector. A strategy of choosing the step size of the correction step is also presented. Application to the problem of $p-$norm Steiner Minimum Trees (SMT) shows that the proposing method is attractive since the computational cost is quite tiny. Comparison with the general extra-gradient method is also provided to show the improvements of the new method.

Keywords: Sum of norms, variational inequality, extra-gradient, Steiner Minimum Trees.