## Shape-Preserving Interpolation and Smoothing for Option Market Implied Volatility

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July 1, 2004

## Abstract:

From several option price observations, interpolation of market implied volatility is often required in practice and empirical study. However, the existing interpolation methods may not satisfy the property that the European call option price function is monotonically decreasing and convex with respect to the strike price. In the paper, a modified convex interpolation method is given for approximating the option price function while explicitly incorporating the shape restrictions. This method can be seen as a continuous state space extension of Jackworth and Rubinstein (1996). By combining the smoothing technique with the shape-preserving interpolation, a nonparametric estimation method is developed for noisy observations. Furthermore, numerical performance of the method is examined. Both Monte Carlo experiments and S&P 500 index option market data analyses show that the method is accurate, flexible and robust.

**Key Words.** Option price function, risk-neutral density, implied volatility, inverse problem, convex interpolation, nonparametric estimation.

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