On control of simple PDEs using MPCC ideas

Michael Hintermüller (Graz), Daniel Ralph (Cambridge) and Stefan Scholtes (Cambridge)

Abstract

We consider the problem involving control (via parameters u) of PDE (in state and adjoint variables y and λ) in the form of an infinite dimensional complementarity problem:

$$\begin{array}{rl} \min\limits_{u,y} & J(u,y) \\ \text{subject to} & -\Delta y - \lambda - f(u) = g, \quad y|_{\partial\Omega} = 0 \\ & y \geq 0 \\ & \lambda \geq 0 \\ & y\lambda = 0, \end{array} \qquad (\mathcal{P})$$

with $y \in H^1(\Omega), \lambda, u \in L^2(\Omega)$. We assume for simplicity that the objective function is convex and that the inequalities and the last equation hold pointwise almost everywhere on the domain Ω.

The starting point for this talk is to take stationarity theory for finite dimensional bilevel programming, namely for Mathematical Programs with Complementarity Constraints, MPCCs (more generally MPs with Equilibrium Constraints), and extend it to the above problem. This allows us to also consider how decomposition methods for MPCCs might relate to algorithms proposed for $\mathcal{P}$ and related problems.