A CBFGS update for reduced Hessian SQP*

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Abstract

In this paper, we present a reduced Hessian SQP method for solving consider equality constrained problem:

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & c(x) = 0, \end{array} \tag{0.1}$$

where $f : R^n \to R$ and $c : R^n \to R^m$ are continuously differentiable functions.

Recently, some authors been renewed interest in analyzing and modifying algorithms for solving large-scale nonlinear optimization problems and have presented and analyzed the reduced Hessian sequential quadratic programming(SQP). This research has been motivated by the fact that there exist many large-scale optimization problems in practice. Meanwhile, those traditional sequential quadratic programming algorithms are efficient only for solving small and medium size nonlinear constrained problems. That reduced Hessian SQP methods allow us to use SQP for significant class of very large problems and that they are more efficient than those traditional SQP methods for solving large-scale constrained optimization problems are in that they require less space for storing the quasi-Newton matrix. As to global convergence of algorithms, the latter usually requires the assumption of positive definiteness for the quasi-Newton matrix while the former doesn't if some practical update techniques are applied.

In this paper, we present a new update criterion, which is called as a cautious BFGS update criterion(CBFGS), to improve the *Nocedal-Overton criterion* and the *positive Curvature criterion* for a reduced Hessian sequential quadratic programming (SQP) method, which is used to solve equality constrained optimization problem and state the corresponding algorithm with this update. Under the assumption of Lipschitz continuity for the gradients of objective and constraint functions, we establish the Global convergence of the algorithm with the CBFGS update criterion without assuming the second-order sufficient condition, which is required for both the *Nocedal-Overton criterion* and the *positive Curvature criterion*. In particular, Only under the second-order sufficient condition at a local optimal solution, the CBFGS update reduces to the ordinary BFGS update and the algorithm converges R-linearly. Numerical experiment results show that the CBFGS update criterion improves the *positive curvature criterion* and the *Nodceal-Overton criterion*.

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