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Continuous-Time Safety-First Asset Allocation

Zhong-Fei Li
Department of Finance, Lingnan (University) College, Sun Yat-Sen University
Guangzhou 510275, People's Republic of China
lnslzf@zsu.edu.cn

Consider a financial market in which $n+1$ assets (or securities) are traded continuously in the time horizon $[0,T]$ and indexed by $i = 0,1,\ldots,n$. One of these assets, say $i = 0$, is the riskless bond, and the other n assets are risky stocks. The horizon $[0,T]$ is partitioned as m subintervals by dates $0 = t_0 < t_1 < t_2 < \ldots < t_m = T$. In each of these subinterval the market is a standard Black-Scholes market and at each of these dates the market may give rise to a jump because of significant information. More precisely, asset i's price process $P_i(t)$ evolves according to the following differential equation

$$dP_0(t) = P_0(t)r(t)dt, \quad P_0(0) = 1,$$

$$dP_i(t) = P_i(t)\left(b_i(t)dt + \sum_{j=1}^{n} \sigma_{ij}(t)dB_j(t)\right), \quad P_i(0) = p_i, \quad i = 1,\ldots,n,$$

where $r(t)$ is the rate of interest satisfying

$$r(t) = r_k \text{ if } t \in [t_{k-1}, t_k), k = 1,2,\ldots,m,$$

$b(t) = (b_1(t),\ldots,b_n(t))'$ is the vector of stock-appreciation rates that satisfies

$$b(t) = b_k \text{ if } t \in [t_{k-1}, t_k), k = 1,2,\ldots,m,$$

$\sigma = (\sigma_{ij})_{n\times n}$ is the matrix of stock-volatilities that satisfies

$$\sigma(t) = \sigma_k \text{ if } t \in [t_{k-1}, t_k), k = 1,2,\ldots,m,$$

and $B(t) = (B_1(t),\ldots,B_n(t))'$ is a standard n-dimensional Brownian motion. Here r_k, b_k and σ_k are all constant in time. Moreover, for simplicity, we assume that σ_k is invertible and that $b_k \geq r\mathbf{1}$ for all k, here $\mathbf{1} = (1,1,\cdots,1)' \in \mathbb{R}^m$.

Using the period-wise constant-rebalanced portfolio investment strategies, which means that fractions of the wealth invested in the assets remain constant in each subinterval $[t_{k-1}, t_k)$, in paper we extend safety-first approach to a continuous-time asset allocation problem, which minimizes the probability that the terminal wealth is below a preselected level. A closed-form explicit expression for the optimal investment strategy is then derived.