Global optimization algorithms and their performance as heuristics

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Abstract:

Branch-and-bound algorithms are the only practical tools for globally minimizing a concave function over a polyhedral set, at least at the present stage. Among others, the rectangular branch-and-bound algorithm is known to solve fairly large scale problems if the objective function is separable into a sum of univariate functions. In other cases, however, we cannot but use the simplicial or conical branch-and-bound algorithm, though they are less efficient.

Any class of branch-and-bound algorithms solves a series of linear relaxed problems, each of which gives a lower bound of a subproblem of the target concave minimization. In the separable case, those linear programs are different just in the objective functions and bounding constraints on some variables. Therefore, we can successively solve them using sensitivity analysis of the simplex algorithm, in a very few pivoting operations per each relaxed problem. In the inseparable case, the relaxed problems are different in the constraint matrices as well, and require much more pivoting operations even if we use sensitivity analysis. Moreover, unlike the separable case, none of them inherits the structure of the target problem. This implies that, even if the target problem has some favourable structures such as network flow, they are of no help in solving the relaxed problems.

In this talk, we propose a simplicial branch-and-bound algorithm for solving inseparable concave minimization problems. The relaxed problems used in this algorithm are different only in the objective functions and inherit the structure of the target problem, though the lower bounds somewhat disimprove. To eliminate this drawback, we introduce a Lagrangian-based procedure for tightening the lower bounds. We report some numerical results, and besides discuss the performance as a heuristic algorithm.