A Global Dual Approach to Constrained Approximation and Optimization

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Abstract:
We study constrained optimization problems and best approximation problems, where the constraint set is given by
$$K:=C\cap \left\{ x\in X:-g(x)\in S\right\}.$$
Here C is a closed convex subset of an infinite dimensional space X, S is a closed convex cone which does not necessarily have non-empty interior, and $g:X\rightarrow Y$ is a continuous S-convex function. Characterizations of solutions to these problems have been of substantial interest. The common approach is to employ a constraint qualification that provides a framework for studying these problems. The well known constraint qualifications are essentially built on Slater's constraint qualification which often restricts applications or are based on local conditions such as the strong conical hull intersection property (CHIP). A major deficiency is the lack of a good global criterion which is less restrictive than the Slater constraint qualification.

In this work we provide a global approach to this problem by presenting a global dual constraint qualification. Implications to semidefinite convex programming and to a central problem in constrained approximation will be discussed. Sequential results avoiding a constraint qualification will also be presented. The powerful conjugate analysis together with ϵ-subdifferential calculus paves the way for achieving the results.

(This is a joint work with N. Dinh, G. M. Lee and H. Mohebi.)