A Path Following Method for Solving a Class of Semi-Infinite Programming Problems

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We consider a class of nonlinear semi-infinite programming problem

$$\min_{x} \; F(x) \quad \text{subject to} \quad \max_{y} f(x,y) \leq 0$$

that can be written in the form

$$\begin{cases} \min\limits_{x,y} \; F(x) \\ \text{subject to} \quad \nabla_y f(x,y) = 0, \\ \qquad\qquad\quad\;\; f(x,y) \leq 0, \end{cases}$$

where $x \in R^n$, $y \in R^m$, $F : R^n \to R$, $f : R^n \times R^m \to R$, and we assume the functions involved are smooth enough. Let us now consider the equality-constrained minimization problem

$$\begin{cases} \min\limits_{x,y,s} \; F(x) - \mu \ln s \\ \text{subject to} \quad \nabla_y f(x,y) = 0, \\ \qquad\qquad\quad\;\; f(x,y) + s = 0 \end{cases}$$

with a logarithmic barrier in its objective, where $s > 0$ is a slack and $\mu > 0$ is a penalty, and define a Lagrangian function by

$$L(x,y,s,\psi,\lambda) := F(x) - \mu \ln s + \psi^T \nabla_y f(x,y) + \lambda\bigl(f(x,y) + s\bigr),$$

where ψ and λ are multipliers for the two equality constraints. For simplicity, the vector $(x,y,s,\psi,\lambda) \in R^{n+2m+2}$ is denoted by X. A necessary optimality condition for the penalized problem is then given by the Lagrange multiplier rule $\nabla L(X) = 0$, and, when μ goes to zero, the solution sequence gives the central path leading to a solution to the original problem.

We will present in this talk a path following algorithm based on successive solution of systems of ordinary differential equations in the form

$$\dot{X} = -M(X)\nabla L(X)$$

for each μ, where $M(X)$ is a matrix valued function and $\dot{X}$ denotes the derivative of X with respect to an independent variable. Convergence analysis will be given through some numerical experiments.

Keywords: semi-infinite programming, nonlinear programming, path following algorithm, ordinary differential equation, stiff system, Newton method.

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