On image space approach to constrained optimization and related topics.

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The purpose of this talk is to give a short account of the Image Space (IS) Analysis approach to Constrained Extremum Problems (CEP) and Variational Inequalities (VI). Even if the study of the properties of the image of a real-valued function is an old one, its application to CEP is a recent one. However, in most cases the properties of the image have not been the scope of the study and their investigation has occurred as an auxiliary step toward other achievements. The traces of IS Analysis can be found in the work of Caratheodory; Bellman, in the 1950s, with his celebrated maximum principle, proposed, for the firs time, to replace the given unknown by a new one which runs in the IS; however, also here, the image is not the main purpose. Only in the late 1960s a and early 1970s, independently each other, some Authors brought explicitly such a study into the field of CEP and, much later, into that of VI. The approach consists in introducing the space, calle IS, where the images of the functions of the given CEP (or VI) run. Then a new problem is defined in the IS, which is equivalent to the given one. In a certain sense, such an approach has some analogies with what happens in the Measure Theory when one goes from Mengoli-Cauchy-Riemann measure to the Lebesgue one. The analysis in the IS must be viewed as a preliminary and auxiliary step - and not as a concurrent of the analysis in the given space - for studying a CEP. If this aspect is understood, then the IS Analysis may be highly fruitful. In fact, in the IS we have a sort of "regularization": the image problem may be convex or continuous or smooth when the given CEP does not enjoys the same property. The present talk aims to give an account of the results on CEP, which have been achieved by means of IS Analysis.