Perfect Duality Theory and Complete Solutions to a Class of Nonconvex Optimization Problems with Applications

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Abstract:
The speaker will present a complete set of solutions for a class of nonconvex optimization problems subjected to inequality constraints. These problems are directly related to numericalization of a large class of semi-linear nonconvex partial differential equations in nonconvex mechanics including phase transitions, chaotic dynamics, bifurcation, nonlinear field theory, and superconductivity. The method used is the so-called canonical dual transformation developed recently. The speaker will show that by the use of this powerful method, these very difficult nonconvex constrained primal problems in n-dimensional space can be converted into an one-dimensional canonical dual problem, i.e. the perfect dual formulation with zero duality gap and without any perturbation. This dual problem can be solved completely. Therefore, a complete set of solutions to the primal problem is obtained. The extremality of these solutions are controlled by the triality theory discovered recently by the speaker. Several examples are illustrated including nonconvex constrained quadratic programming. Results shown that these problems can be solved completely to obtain all KKT points and global minimizers. Also certain NP-hard problems in n-dimensional space can be converted into a concave maximization dual problem in lower dimensional space. Moreover, a complete set of solutions for quadratic programming over a sphere is also presented as a by-product. A powerful triality algorithm is presented, and several examples are illustrated.