Optimal Replacement of a System in a Semi-Markov Environment

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Optimal replacement problem is an interesting research area for a long time. It considers a system that will deteriorate and thus should be replaced by a new one when it deteriorated too bad. There are two types of the deterioration considered in reliability literature, one is due to the operation of the system while the other is caused by influence of the environment to the system. The influence occurs intermittently and is often described by a Poisson process. But in general these two types of deterioration are considered separately in literature.

This paper investigates an optimal replacement problem of a system in a semi-Markov environment. The system itself deteriorates according to a semi-Markov process, and is further influenced by its environment, which changes according to a semi-Markov process. Each change of the environment's state will change the parameters modelling the system and also cause damage on the system.

The system considered here is as follows:

(1) The system is in a semi-Markov environment $\{(J_n, L_n), n \geq 0\}$ with a kernel $G_{kk'}(t)$ on a set K of countable environment states. Let $\psi_{kk'} = G_{kk'}(\infty)$ and $G_k(t) = \sum_{k'} G_{kk'}(t)$.

(2) During an environment state k, the system itself operates according to a semi-Markov process with the kernel $\{P_{ij}^k(t), i, j \in S\}$ and countable state set $S = \{0, 1, 2, \ldots\}$, where the state 0 represents a new system, and states $1, 2, \ldots$ represent the different degrees of deterioration of the system, and the bigger the value, the more serious the deterioration. Let $p_{ij}^k = P_{ij}^k(\infty)$, $T_{ij}^k(t) = P_{ij}^k(t)/p_{ij}^k$, and $T_i^k(t) = \sum_j P_{ij}^k(t)$.

(3) Suppose that the environment is in state k, then one of the following two actions can be chosen if the system state transfers to i:

- operate continually the system (denoted by O) with a cost rate $h^k(i)$;
- replace the system by a new one (denoted by R) with a cost rate $c^k(i)$, and the time of the replacement is assumed to be a random variable with distribution function $F(t)$, and the state after replacement will be 0.

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(4) When the environment state changes from k to k', if action O is being chosen then the system state will change immediately according to a probability q_{ij}^k and an instantaneous cost $R^k(i,0)$ occurs; while if action R is chosen then the replacement is immediately completed and an instantaneous cost $R^k(i,R)$ occurs.

(5) The objective is to minimize the expected discounted total costs with discount factor $\alpha > 0$.

Such a system is modelled by a semi-Markov decision processes (SMDP) in a semi-Markov environment.

For minimizing the discounted total costs, we show the existence of an optimal control limit policy for both finite and infinite horizon. A special case of Markov environment is discussed and better results are obtained, and the state space is reduced to be finite. Finally, it is shown that the same results is true for the instantaneous replacement.