On the Convergence of a Trust Region Method for Solving Constrained Nonlinear Equations

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This paper presents a trust region method for solving a constrained system of nonlinear equation

$$F(x) = 0, \ x \in \Omega, \tag{0.1}$$

where $\Omega \subset R^n$ is a nonempty closed convex set and $F: \ R^n \supset U \to R^n$ is defined on the open set $\mathbf{U}$ containing Ω and is continuously differentiable.

The main properties of new method proposed in this paper is twofold: one is its choice of search direction, which is an optimal combination of projected gradient direction and projected trust region direction. By using this technique, we overcome the possible non-convergence of the projected Newton direction. Other is its strong convergence. We prove the global convergence of new algorithm. Especially for its local convergence, without nonsingular condition of Jacboian matrix of $F(x)$ at the solution, the new method enjoys locally quadratic convergence under local error bounded condition of $F(x)$. This result extents the conclusion of Yamashita and Fukushima, Fan and Yuan for unconstrained nonlinear equations to the constrained system of nonlinear equations (0.1).