

# An Approach to Nonlinear Robust Control and Its Application to Power System

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## Abstract

### ✧ The Approach

As we know, the control laws being searched to solve the nonlinear robust control problem rely upon the solutions of so-called Hamilton-Jacobi-Isaacs inequality<sup>[1]</sup>. However, it is a nonlinear partial differential one, there is no general approach to obtain the solution of it.

Combining the state feedback exact linearization method with linear robust control principle, an applicable approach to robust control of affine nonlinear systems is introduced<sup>[2]</sup> in this discussion.

Consider an affine nonlinear system in the form as

$$\begin{aligned}\dot{x}(t) &= f(x) + g_1(x)w + g_2(x)u \\ y(t) &= h(x)\end{aligned}\tag{1}$$

where  $x \in R^n, u \in R$  and  $y \in R$  are state, control and regulation output vectors, respectively;  $w \in R^2$  denotes disturbance;  $f, g_1, g_2$  and  $h(x)$  ( $f(0) = 0, h(0) = 0$ ) are smooth vector fields.

The problem is to find a small enough number  $\gamma^* > 0$ , and a control strategy  $u = u^*(x)$ , such that

$$\int_0^T (\|y\|^2 + \|u\|^2) dt \leq \gamma^2 \int_0^T \|w\|^2 dt \quad \forall T > 0$$

holds and the closed-loop system is asymptotically stable provided  $w = 0$

By means of coordinate mapping and state feedback

$$z = \phi(x) \quad v = \alpha(x) + \beta(x)u \quad (2)$$

the affine system

$$\dot{x} = f(x) + g_2(x)u \quad (3)$$

$$y = h(x)$$

can be transformed into a linear system in the form as <sup>[2,3,5]</sup>

$$\begin{aligned} \dot{z} &= Az = B_2 v \\ y_z &= Cz \end{aligned} \quad (4)$$

Then the system (1) can be written as

$$\begin{aligned} \dot{z} &= Az + \frac{\partial \phi(x)}{\partial x} g_1(x)w + B^2 v \\ y_z &= Cz \end{aligned} \quad (5)$$

Sct  $\bar{w} = \frac{\partial \phi}{\partial x} g_1(x)$ , then (5) can be written as

$$\begin{aligned} \dot{z} &= Az + B_1 w + B_2 v \\ y_2 &= Cz \end{aligned} \quad (6)$$

Now, we can design an optimal controller for system (6) according to the linear robust control principle. The problem has a solution if the following Riccati inequality <sup>[5]</sup>

$$A^T P + PA + \frac{1}{\gamma^2} P B_1 B_1^T P - P B_2 B_2^T P + C^T C < 0 \quad (7)$$

has a nonnegative solution  $P^*$ . Then, the optimal control strategy  $v^*$  is

$$v^* = -B_2^T P^* z \quad (8)$$

and the worst possible disturbance  $\bar{w}$  in (6) is

$$\bar{w}^* = \frac{1}{\gamma^2} B_1^T P^* z \quad (9)$$

According to (2) and (9), the control law  $u^*$  in coordinate  $x$  can be written as

$$u^* = -b^{-1}(x)(a(x) + B_2^T P^* \phi(x)) \quad (10)$$

where  $u^*$  stands for the nonlinear robust control law for the original system in Eq. (1)

✧ **Nonlinear Robust Excitation Control of Power Systems** <sup>[4, 5]</sup>

The design approach mentioned above based on feedback linearization can be applied to design robust excitation control for multi-machine systems.

The modeling, the approach to optimal robust excitation control, the control strategy and the simulation results are discussed demonstrated.

**REFERENCES:**

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