AN TYPE OF ABS ALGORITHM FOR SOLVING SINGULAR NONLINEAR SYSTEMS WITH RANK DEFECTS(2)

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1.abstract

Consider the following nonlinear system

$$F(x) = 0 \tag{1}$$

where $x \in R^n$, $F(x) = (f_1(x), f_2(x), \ldots, f_n(x))^T$. We assume that there exists a solution x^* of (1), $F(x^*) = 0$, F'' is Lipschitzian around x^* and furthermore that

$$\begin{cases} \mathrm{rank}(F'(x^*)) = n - r, r > 1 \\ \text{Let}: \tau_{i,j} = v_i^T F''(x^*) u_i u_j, \text{and} g_{i*}^T = v_i^T F''(x^*) u_i u_i \\ \det(\tau_{i,j}) \neq 0, g_{i*}^T \neq 0 \end{cases} \tag{2}$$

where $u_1, u_2, \ldots, u_r$ and $v_1, v_2, \ldots, v_r$ are arbitrary orthogonal vector sets such that

$$\begin{aligned} \mathrm{Null}(F'(x^*)) &= \mathrm{span}\{u_1, u_2, \ldots, u_r\} \\ \mathrm{Null}(F'(x^*)^T) &= \mathrm{span}\{v_1, v_2, \ldots, v_r\} \end{aligned} \tag{3}$$

And Let

$$N = (u_1, u_2, \ldots, u_r), V = (v_1, v_2, \ldots, v_r)$$

Taking $r \times r$ matrix P, Q, $Q \approx V$, $P \approx U$, Let

$$(P^T U)^{-1} = (s_1, s_2, \ldots, s_r), (Q^T V)^{-1} = (h_1, h_2, \ldots, h_r)$$

We constitute a new function

$$T(x) = (F(x), w_1(x), w_2(x), \ldots, w_r(x))^T$$

,where $w_i(x) = h_i^T V^T F'(x) U s_i$, then $T(x^*) = 0$ and $T'(x^*)$ is of full rank.
By combining the discreted ABS algorithm and extending ideas of Hoy and Schwetlick presented in 1990,we have established a modified ABS method and discussed its Q-superlinear convergence.
[Notice]: for a particular situation of $\mathrm{rank}(F'(x^*)) = n - 1$, we have published the corresponding result in Korean J.Comput.Appl.Math.(2002),Vol.9,No.1.