AN MODIFIED ABS ALGORITHM FOR SOLVING SINGULAR NONLINEAR SYSTEMS WITH RANK ONE DEFECT(1)

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1.abstract

Consider the following nonlinear system

$$F(x) = 0 \tag{1}$$

where $x \in R^n$, $F(x) = (f_1(x); f_2(x); \ldots; f_n(x))^T$. We assume that there exists a solution x^* of (1), $F(x^*) = 0$, F'' is Lipschitzian around x^* and furthermore that

$$\left\{ \begin{array}{ll} \mathrm{rank}(F'(x^*)) = n - 1 & \\ \nabla f_i(x^*) \neq 0 & i = 1; 2; \ldots n; \end{array} \right. \tag{2}$$

Assume c^* is the unique solution of the following equation (3)based on the processing of pivoting Gauss elimination with a component of c^* obtaining 1

$$J_{n-1}(x^*)c^* = \begin{pmatrix} \nabla f_1(x^*) \\ \nabla f_2(x^*) \\ \vdots \\ \vdots \\ \vdots \\ \nabla f_{n-1}(x^*) \end{pmatrix} * c^* = 0 \tag{3}$$

Letting: $\hat{f_n}(x) = f_n(G^*x + b^*)$,where

$$(G^*)^T = \frac{c^* \nabla f_n^T(x^*)}{\| \nabla f_n^T(x^*) \|^2}; \quad b^* = x^* - G^*x^*$$

Let $T(x) = (f_1(x); f_2(x); \ldots \hat{f_n}(x))^T$. Then $T(x^*) = 0$ and $T'(x^*)$ is of full rank. By combining the discreted ABS algorithm and the idea of rotating transforming ,we have established a modified ABS method and discussed its Q-superlinear convergence. In practical, at the process of iterating we will generate two sequence $G^{(k)}$ and $b^{(k)}$ such that

$$G^{(k)} \to G^*; \; ; \; b^{(k)} \to b^*:$$

In algorithm ,we hope to substitute $\hat{f_n}(x)$ by $\hat{f_n^{(k)}}(x) = f_n(G^{(k)}x + b^{(k)})$.