Quadratic Stability and Stabilization of Switching Linear Systems

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Abstract

In nowadays the problems about switching systems become a hot topic in control community. The model comes from robot control, biological system, traffic control, power system, aircraft etc. Even if the animal and human behaviors are switching systems. This presentation concerns the quadratic stable and stabilization of switching systems. The problems of stability, stabilization and stabilization via control and switching are considered. The followings are the main results obtained correspondingly.

A switching linear system is defined as

$$\dot{x} = A_{\sigma(x,t)}x, \quad x \in \mathbb{R}^n, \tag{1}$$

where $\sigma : \mathbb{R}^n \times [0,\infty) \to \Lambda$, and $\Lambda = \{1,\cdots,N\}$ is a finite set. A switching linear control system is defined as

$$\dot{x} = A_{\sigma(x,t)}x + B_{\sigma(x,t)}u, \quad x \in \mathbb{R}^n, \;\; u \in \mathbb{R}^m. \tag{2}$$

Definition 1. *System (1) is quadratically stable if there exists a quadratic Lyapunov function $V = x^T P x$ with $P > 0$ such that*

$$PA_i + A_i^T P < 0, \quad i = 1,\cdots,N. \tag{3}$$

In the following let $L = \frac{n(n+1)}{2}$, $t = (t_1,\cdots,t_L)$.

Theorem 1. *The system (1) is quadratically stable, iff*

$$\begin{array}{c}\int_0^{2\pi} dt_1 \cdots \int_0^{2\pi} dt_L \int_{L_1(t)}^{V_1(t)} dx_1 \int_{L_2(t,x_1)}^{V_2(t,x_1)} dx_2 \cdots \\ \cdots \int_{L_{n-2}(t,x_1,\cdots,x_{n-3})}^{V_{n-2}(t,x_1,\cdots,x_{n-3})} dx_{n-2} \int_{L_{n-1}(t,x_1,\cdots,x_{n-2})}^{V_{n-1}(t,x_1,\cdots,x_{n-2})} dx_{n-1} > 0.\end{array} \tag{4}$$

In the above theorem $V_k(t,x_1,\cdots,x_{k-1})$ and $L_k(t,x_1,\cdots,x_{k-1})$, $k = 1,\cdots,n-1$ are easily computable functions.

Definition 2. *The system (2) is quadratically stabilizable if there exist feedback K_i, $i = 1,\cdots,N$, and a quadratic Lyapunov function $V = x^T P x$ with $P > 0$ such that*

$$P(A_i + K_i B_i) + (A_i + K_i B_i)^T P < 0, \quad i = 1,\cdots,N. \tag{5}$$

Theorem 2. *Assume $n = 2$ and $m = 1$. The system (2) is quadratically stabilizable, iff the following Liner algebraic inequality has solution:*

$$Lx > 0, \tag{6}$$

where L is an easily computable matrix.

Definition 3. *The system (1) is quadratically stable via switching if there exist a switching rule $\sigma(x)$ and a quadratic Lyapunov function $V = x^T Px$ with $P > 0$ such that*

$$PA_{\sigma(x)} + A^T_{\sigma(x)} P < 0; \tag{7}$$

The system (2) is quadratically stabilizable via switching if there exist feedback K_i, $i = 1, \cdots, N$, a switching rule $\sigma(x)$, and a quadratic Lyapunov function $V = x^T Px$ with $P > 0$ such that

$$P(A_{\sigma(x)} + K_{\sigma(x)} B_{\sigma(x)}) + (A_{\sigma(x)} + K_{\sigma(x)} B_{\sigma(x)})^T P < 0. \tag{8}$$

Define

$$V = \langle A_1, \cdots, A_N | B_1, \cdots, B_N \rangle .$$

With respect to this V, the system (2) can be expressed as

$$\begin{pmatrix} \dot{x}^1 \\ \dot{x}^2 \end{pmatrix} = \begin{pmatrix} A^{11}_{\sigma(x,t)} & A^{12}_{\sigma(x,t)} \\ 0 & A^{22}_{\sigma(x,t)} \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} + \begin{pmatrix} B^{11}_{\sigma(x,t)} \\ 0 \end{pmatrix} u. \tag{9}$$

Theorem 3. *The system (9) is quadratically stabilizable via switching, if*

$$\dot{x}^1 = A^{11}_{\sigma(x,t)} + B^{11}_{\sigma(x,t)} u$$

is quadratically stabilizable, and

$$\dot{x}^2 = A^{22}_{\sigma(x,t)}$$

is quadratically stable via switching.