Relations Between Almost Pareto and Almost Kuhn-Tucker in Vector Optimization

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Abstract. It is well known that for a lower bounded differentiable function $f : X \to \mathbb{R}$, where X is a Banach space, at a point $a \in X$ such that $f(a)$ is near $\inf f(X)$ the gradient is not necessarily almost zero. A very useful answer to the question : "what can be said about such a point?" is given by the famous Ekeland variational principle.

Conversely, if in addition f is convex, for a point $a \in X$ such that $\nabla f(a)$ is near zero (i.e. a is an almost stationary point) we cannot be sure that $f(a)$ is close to $\inf f(X)$, unless f is asymptotically well behaved (a.w.b.). Equivalent conditions, based especially on the sublevel sets, for a convex function to be a.w.b. have been given by Auslander, Crouzeix and others.

Our aim is to present some generalizations of the above results to *vector* functions, i.e. $f : X \to Y$ where Y is a (partially) ordered Banach space. Thus "$f(a)$ near $\inf f(X)$" becomes "a is an almost Pareto (efficient) point" which means that "$f(a)$ is close to the *infimal set"* (which is no more a singleton, it may be even unbounded!). An "almost stationary point" becomes an "almost *scalarly stationary (or Kuhn-Tucker) point",* i.e. is an almost stationary point for some *scalarized* function $\lambda \circ f$ with λ a positive element of the topological dual of Y.

Thus we will present some metrically consistent vector variational principles, showing that near an almost Pareto point, we can find another almost Pareto point which is at the same time an almost Kuhn-Tucker point.

Conversely, we will present several necessary and/or sufficient conditions for a convex vector function to have the property that an almost Kuhn-Tucker point is an almost Pareto one.