A Solution to Hamilton-Jacobi Equation by Neural Networks and Optimal State Feedback Control Law of Nonlinear Systems

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Abstract

This paper is concerned with a state feedback controller using neural networks for nonlinear optimal regulator problem. Nonlinear optimal feedback control law can be synthesized by solving the Hamilton-Jacobi partial differential equation with three layered neural networks. The Hamilton-Jacobi equation solves the value function $V^o(\mathbf{x})$ from which the optimal feedback law can be synthesized. To obtain an approximate solution $V^N(\mathbf{x})$ to the Hamilton-Jacobi equation by the network, we solve an optimization problem which determines connection weights in the neural network. Gradient functions with respect to the connection weights are calculated explicitly by applying the Lagrange multiplier method and used in the learning algorithm of the network.

The method is relatively easy when applied to affine nonlinear systems, since we have only to approximate the value function in the H-J equation with the neural network such that an error of the H-J equation goes to zero. When applied to general nonlinear systems, however, its computation becomes very complex, because we must approximate both the value function and control inputs with neural networks such that necessary optimality conditions for the control inputs are satisfied. Nevertheless, since we can calculate the gradient functions by the Lagrange multiplier method, the learning of neural networks is carried out very efficiently and systematically.

It is noted that a solution to the H-J equation is not necessarily unique and so an approximate solution $V^N(\mathbf{x})$ by the network is not guaranteed to be the true value function $V^o(\mathbf{x})$. This difficulty is caused by the fact that the H-J equation is only a necessary condition for optimality. Hence we propose a device for learning of the network such that an approximate solution $V^N(\mathbf{x})$ converges to the true value function $V^o(\mathbf{x})$, using a stabilizing solution of the Riccati equation for the linearized LQ regulator problem. Actually we are interested in optimal control that the closed-loop system is asymptotically stable and so, we try to seek a stabilizing optimal solution to the H-J equation. Under some appropriate assumptions such as detectability, we can make the approximate solution $V^N(\mathbf{x})$ converge to such a solution by considering the relation $\nabla^2 V^N(\mathbf{0}) = P^-$, where P^- denotes a stabilizing solution to the Riccati equation for LQ optimal regulator problem corresponding to the original nonlinear one. Consequently the nonlinear optimal feedback control law is obtained from $V^N(\mathbf{x})$ such that the closed-loop system is asymptotically stable and optimal.

The effectiveness of the proposed method was confirmed from simulation results for various plants consisting of both affine and general nonlinear systems.