Optimal Collision Avoidance Using Nonlinear Programming

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Recently there have been several studies dealing with the use of discretization and nonlinear programming in solving optimal control problems (see e.g. [1], [2] and for applications in pursuit evasion games [3]). The original problem is approximated by discretizing the dynamics of the system under consideration in time and describing them with a set of equality and inequality constraints. The performance index of the original problem is optimized subject to these constraints using an appropriate nonlinear programming algorithm. The discretization can be carried out in a number of ways (see e.g. [4]). Methods using the above approach are usually referred to as direct methods, since they do not explicitly deal with the necessary conditions of optimality. As increasingly complex nonlinear dynamic systems are considered, the performance of the optimization method has a greater importance.

The problem considered in this presentation represents an encounter between an aircraft and a missile where the aircraft tries to avoid the missile. The missile uses a feedback guidance law which is assumed to be known. The missile's trajectory is thus defined by its initial state and the controls of the aircraft. Hence the problem of missile avoidance can be formulated as an optimal control problem with the controls of the aircraft as decision variables. We consider the final phase of the encounter, also known as the endgame, where the missile is initially flying with a supersonic velocity on a collision course within some kilometers from the target aircraft. Both the missile and the aircraft move in three dimensions. The topic has been studied extensively with various simplified models (see refs. [5]-[9]). Unlike these studies, we employ a model that takes into account the finite angular velocities of the aircraft as well as the constraints regarding the angle of attack and its change rate. In this study the objective of the aircraft is to maximize the minimum distance, also known as the miss distance, to the missile. The moment of minimum distance is given by a geometric condition that defines the otherwise free end time and removes the bilevel structure of the problem.

The optimal control problem is parameterized using direct multiple shooting, which is derived from direct shooting, also called control parameterization (e.g. [4]). The solution interval is divided into subintervals, and the control as well as the state trajectories are required to be continuous over these intervals. Inside the intervals, the controls are approximated piecewise linearly, and the states are integrated. To treat the control rate

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constraints we propose an approach where these constraints are taken into account by di®erential inclusions [10]. Traditionally these constraints are treated by introducing the control rates as new controls and transforming the original controls into state variables. This turns all the original control constraints into state constraints that are generally quite di±cult to handle. In the di®erential inclusions approach we require that the values of the control variables at the next discretization gridpoint must be attainable from the values of the control variables at the previous gridpoint, taking into account the limits of the control rate. The control rates are approximated linearly. The equality constraints for matching the states across the intervals as well as the control constraints depend at most on two consecutive discretized states and the controls between them. Thus the Jacobian of the constraints has a sparse banded structure. The resulting parameter optimization problem is solved using an implementation of sparse sequential quadratic programming.

The purpose of the division into subintervals is to improve the convergence of the optimization. The aim is to suppress the (large) e®ect of controls in the beginning of the solution interval on the solution trajectory in the end of the interval. In the present problem the improvement is important, since the geometric condition indicating the minimum distance is a di±cult constraint from the optimization point of view.

Examples with realistic aircraft and missile drag, thrust, and constraint data are computed. The solutions are compared with those obtained with a point mass model, and the results are discussed.

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