A NEW CLASS OF PROXIMAL ALGORITHMS FOR THE NONLINEAR COMPLEMENTARITY PROBLEM

P. R. Oliveira[1] and G. J. P. da Silva[2]
PESC/COPPE - Federal University of Rio de Janeiro
[1]poliveir@cos.ufrj.br [2]geci@cos.ufrj.br

Abstract

We consider the nonlinear complementarity problem,

$$\mathrm{NCP}(F): \text{ to obtain } x \in \mathbb{R}^n;\ x \geq 0;\ F(x) \geq 0;\ x^T F(x) = 0;$$

where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a C^1 function. Facchinei and Kanzow, and, Yamashita, Imai and Fukushima, both in 1999, proposed algorithms that suppose that F is a P_0 function. F is a P_0 function, if it verifies that, for all $x;\ y \in \mathbb{R}^n$; with $x \neq y$; there exists some index $i = i(x; y)$ such that the following facts are simultaneously true: $x_i \neq y_i$, and $(x_i - y_i)(F_i(x) - F_i(y)) \geq 0$: Now, we propose, for P_0 functions, a sequence of proximal regularizations, defined through $F^k(x) := F(x) + c_k \left(X^k\right)^{-r}\left(x - x^k\right)$; where $x^k > 0$ and $c_k \in \mathbb{R}_{++}$ are known, $\left(X^k\right)^{-r} = \mathrm{diag}\left\{\left(x_1^k\right)^{-r}; \ldots; \left(x_n^k\right)^{-r}\right\}$, and $r \geq 1$. As a consequence of that formulation, we are able to show that F^k; strongerly than F, is a P function, that is, it verifies the inequality $\max\{(x_i - y_i)(F_i(x) - F_i(y));\ i = 1; \ldots; n\} > 0$; for all $x;\ y \in \mathbb{R}^n$; $x \neq y$: This fact permits us to show the important result that $\mathrm{NCP}\left(F^k\right)$ has a unique solution, for each k. Now, we can describe the algorithm. First define, successively, using the Fischer-Burmeister function, $\varphi(a; b) := \sqrt{a^2 + b^2} - a - b$; and $\Psi^k(x) := \sum_{i=1}^n \varphi\left(x_i; F_i^k(x)\right)^2 / 2$.

Initialization: choose $c_0 > 0$; $\delta_0 \in (0; 1)$ and $x^0 \in \mathbb{R}^n_{++}$. Set $k = 0$:

Step 1: choose $x > 0$, such that $\Psi^k\left(x^{k+1}\right) \leq \delta_k^2$:

Step 2: choose $c_{k+1} \in (0; c_k)$ and $\delta_{k+1} \in (0; \delta_k)$: Update k, return to Step 1.

The main result is: Let's suppose that F is a P_0 function and the solution set of NCP(F) is nonempty and bounded. Also suppose that the following hypothesis about the sequence $\{c_k\}$ are verified: $c_k \left(X^k\right)^{-r}\left(x^{k+1} - x^k\right) \to 0$; if $\left\{x^k\right\}$ is bounded, or, $c_k \left(X^k\right)^{-r} \to 0$ if $\left\{x^k\right\}$ is unbounded. Then, letting $\delta_k \to 0$; we have $\{x_k\}$ bounded, and any accumulation point of $\{x_k\}$ is a solution of NCP(F):

Some improvements on the theoretical results, and extensions are under consideration.