

Analysis of Two Dimensional Non Convex Variational Problems by The Method of Moments

**Contributed Lecture for the 34th International Workshop on
Optimisation and Control**

Erice, July 8-17, 2001

R. Meziat

Edificio Melchor Cano, 16071, Cuenca, Spain

Universidad de Castilla La Mancha

rmeziat@arq-cu.uclm.es

Abstract

For Non Convex Variational Problems it is not clear if they have or have not a solution, because it is not possible to apply on them the classical tools of the Direct Method of the Calculus of Variations. In addition, these problems are linked with highly oscillatory phenomena which arise from the non-convexity in the integrand. For instance consider the functional

$$I(u) = \iint_{\Omega} \left\{ (1 - u_x^2)^2 + u_y^2 + u(x, y)^2 \right\} dx dy$$

which describe the energy balance on an elastic plate. Under certain boundary conditions this functional may lack of a minimum in a current function space, however its minimizing sequences exhibit an oscillatory behaviour that represents the kind of solid phase that may appear on the body. To analyse this situation we must use a relaxed formulation in Young measures; that is an extremely complex optimisation problem defined in parametrized measures. In order to simplify it, we can use the algebraic moments of the parametrized measures involved, so we obtain a new optimisation problem that can be solved by standard optimisation techniques. I stress that the new formulation becomes a particular problem in semi-definite programming. Moreover, the solution of the relaxed problem in Young measures can be recovered using well known results on Moments Theory, so we can predict the behaviour of the minimizing sequences of the original non convex variational problem. This method has been successful to treat non convex, one dimensional, scalar variational problems when the integrand has a particular polynomial structure. In this lecture I will present the extension for two and higher dimensions of the Method of Moments for Non Convex Variational Problems.