

Distributed computation of incentive Stackelberg solutions

Harri Ehtamo, Mitri Kitti^a, Raimo P. Hämmäläinen
Systems Analysis Laboratory,
Helsinki University of Technology
P.O. Box 1100, 02015 HUT, FINLAND

Decision problems involving multiple decision-makers that act in a predetermined order are common in many real world problems. Such situations can be considered as Stackelberg games, i.e., games where one player (the leader) announces his decision (action) to other players (followers) who take this decision into account when designing their reactions. Applications of Stackelberg games include, e.g., electricity pricing, oligopolistic market analysis and resource allocation. For an application in a τ -series management case see, e.g., [5].

In the case where the followers act first and the leader can observe the followers' actions, the leader can give his announcement or strategy as a function of these actions, i.e., in a form of an incentive. Such Stackelberg games are called incentive Stackelberg games. The most favorable incentive policy for the leader would be such that the followers' best responses lead to the optimal outcome of the game for the leader. For these formulations see [8] and [2].

Distributed computational methods are based on solving the original problem by decomposing it into subproblems. For a game problem each subproblem corresponds to one or several players. We shall present a new distributed method for finding a $\pm$ ne incentive solutions in incentive Stackelberg games. The method requires only the followers' optimal reactions, i.e., the leader may compute his strategy without explicit knowledge on the followers' cost functions when the game is played repeatedly. Another approach, proposed in [7], is to use genetic algorithms in Stackelberg games to obtain a distributed scheme.

We concentrate in a two-player case, where the $\pm$ ne incentive strategy, parameterized by two vectors, defines a constraint for the follower's optimization problem. The follower optimizes his objective function and gives his response to the leader who updates the incentive accordingly. The updating procedure is based on solving a nonlinear system of equations iteratively either by using fixed-point iteration or quasi-Newton methods.

We present two numerical examples to show the use of the method. The first example is a two-player quadratic case. In the second example we study the use of the algorithm in a situation of two competing firms and a government coordinating them, see [6]. These examples show that the distributed method developed here is effective and has promising convergence properties. Finally, we briefly discuss computationally similar problems where linear constraints are used in coordination. The most famous of such problems is the price coordination problem in microeconomic theory [1]. Another related problem arises in generating Pareto solutions in two-party negotiations by adjusting artificial constraints, see [4].

^aCorresponding author, e-mail: mitri.kitti@hut.

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