Existence in Vector Convex Optimization and Optimality Conditions in Non-Convex Set-Valued Minimization

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Abstract: In the first part of the lecture, we discuss the following problem: given a closed convex subset K in $\Re^n$; a vector function $F : K \times K \to \Re^m$; a closed convex (not necessarily pointed) cone $P(x)$ in $\Re^m$ with non-empty interior, int $P(x)$, various existence results to the problem

$$\text{find } \bar{x} \in K \text{ such that } F(\bar{x}, y) \notin -\text{int } P(\bar{x}) \quad \forall\, y \in K,$$

under the $P(x)$-convexity/lower semicontinuity of $F(x, \cdot)$ and pseudomonotonicity on F. Moreover, under a stronger pseudomonotonicity assumption on F (which reduces to the previous one in case $m = 1$), some characterizations of the non-emptiness of the solution set are given. Also, several alternative necessary and/or sufficient conditions for the solution set to be nonempty and compact are presented. However, the solution set fails to be convex in general. The classical case $P(x) = \Re^m_+$ is specially discussed by assuming semi-strict quasiconvexity. The results are then applied to vector variational inequalities and minimization problems. Our approach is based upon the computing of certain cones containing particular recession directions of K and F. These results were obtained in a joint work with Fernando Flores-Bazán.

In the second part, we establish an optimality condition which is both necessary and sufficient for optimality in non-convex set-valued minimization problems in normed vector spaces. This will be carried out by using the closed radial cone. Such a condition subsumes various necessary and /or sufficient conditions found in the literature for single-valued convex/non-convex mappings.

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