GENERALIZED MONOTONE VECTOR VARIATIONAL INEQUALITIES OVER PRODUCT SETS

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Abstract.

In recent years there has been an increasing interest in vector variational inequality (VVI); mainly this study in ¯nite-dimensional Euclidean spaces has been ¯rst introduced in [1]. Recently, the equivalence between a VVI and a vector optimization problem (VOP) as well the equivalence between VVI problem a vector complementary problem (VCP) have been intensively analysed in [2] { [5]. Various classes of vector variational inequalities were studied both in ¯nite- and in in¯nite-dimensional spaces; see e.g. [9]. There are several generalizations of VVI problems in which the cost mapping is supposed to be set-valued (GVVI); see e.g. [6, 7]. In this paper, we present an extension of several existence results for generalized vector variational inequalities to set-valued mappings over product sets in a topological vector space setting. It is well known that a number of equilibrium type problems arising in Economics, Game Theory and Transportation have a decomposable structure, namely, they can be formulated as vector variational inequalities over Cartesian product sets; see e.g. [10]{ [12]. At the same time, most existence results for such problems are based on the known ¯xed point techniques, which require either the feasible set (otherwise, the corresponding subset associated to a coercivity condition) be compact in the strong topology or the cost mapping possess certain continuity type properties with respect to the weak topology; see e.g. [10, 13]. Usually, to essentially weaken these assumptions one make use of the Ky Fan Lemma [8] together with certain monotonicity type properties regardless the decomposable structure of VI; see e.g. [9, 14]. By employing new

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monotonicity type concepts which are adjusted for a decomposable structure of the initial problem, Konnov in [15] proved existence results for scalar variational inequalities under weaker assumptions. The new relative monotonicity concepts can be regarded as intermediate ones between the standard monotonicity and P-mapping notions.

Being based upon this approach, we [16] strengthen existence results which were based on the usual monotonicity type assumptions for GVVIs in the particular case of a product set with a finite number of indices. Now we present several new results which generalize and strengthen those in [16]. In particular, they include the case of countable and uncountable set of indices.

Key words: Vector variational inequalities, set-valued mappings, product sets, relative monotonicity, existence results.

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