DEPARTMENT OF APPLIED MATHEMATICS
應用數學系

The Hong Kong Polytechnic University
Department of Applied Mathematics

Colloquium

Optimal basis algorithm and its application to interpolation and matrix scaling

by

Professor Oleg Burdakov

Linköping University

Abstract

We present the optimal basis (OB) problem and the OB algorithm that we proposed in BIT (1997) **37**, 591-599. The OB problem is formulated as follows. Given $m+1$ points $\{x_i\}_0^m$ in R^n which generate an m-dimensional linear manifold, construct for this manifold a maximally linearly independent basis that consists of vectors of the form $x_i - x_j$. This problem is present in, e.g., stable variants of the secant and interpolation methods, where it is required to approximate the Jacobian matrix f' of a nonlinear mapping f by using values of f computed at $m + 1$ points. In this case, it is also desirable to have a combination of finite differences with maximal linear independence. As a natural measure of linear independence, we consider the Hadamard condition number which is minimized to find an optimal combination of m pairs $\{x_i, x_j\}$ that defines the optimal basis. This problem is not NP–hard, but can be reduced to the minimum spanning tree problem, which is solved by the greedy algorithm in $O(m^2)$ time. The complexity of this reduction is equivalent to one $m\times n$ matrix–matrix multiplication, and according to the Coppersmith–Winograd estimate, is below $O(n^{2.376})$ for $m = n$. We discuss possible applications of the OB algorithm for constructing simple non-diagonal prescaling procedures for iterative linear algebra solvers.

Date : 20 November, 2018 (Tuesday)
Time : 11:00a.m. – 12:00noon
Venue : TU801, The Hong Kong Polytechnic University

***** ALL ARE WELCOME *****