



**The Hong Kong Polytechnic University
Department of Applied Mathematics**

**Seminar
on**

**Solution to an optimal control problem by
Krotov method**

by

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Abstract

In this talk, we study the problem on how to get an analytic solution to an optimal control problem. By use of Krotov extension method we find a new performance index by constructing an auxiliary function to get an extension problem which is equivalent to the original problem. By canonical differential flow we solve some convex and non-convex global optimization problems which is induced from the optimal control problem. As an example, we consider the following optimal control problem:

$$\begin{aligned} P_1 \quad & \min J(u(\cdot)) = \frac{1}{2} \int_{t_0}^{t_f} u^T(t) U u(t) dt \\ \text{s.t.} \quad & \dot{x}(t) = Ax(t) + Bu(t), \quad t \in [t_0, t_f] \\ & x(t_0) = x_0, x(t_f) = x_1, \\ & x_0, x_1 \in R^n, \\ & u(t) \in \Sigma = \{u \mid u^T u \leq 1\}, \quad t \in [t_0, t_f]. \end{aligned} \tag{1}$$

For the problem above, an analytic expression of the optimal control is when $0 \leq t < 1 + \ln \hat{c}$,

$$\hat{u}(t) = [e - 1 - (\sqrt{e(e-2)})] e^{1-t-1} \hat{c} e^{1-t} = 1.$$

and when $1 + \ln \hat{c} \leq t \leq 1$,

$$\hat{u}(t) = [1 + 0]^{-1} \hat{c} e^{1-t} = [e - 1 - (\sqrt{e(e-2)})] e^{1-t}.$$

Date : 4 October 2011 (Tuesday)
Time : 11:00 am – 12:00 noon
**Venue : Departmental Conference Room HJ610
The Hong Kong Polytechnic University**

***** ALL ARE WELCOME *****