Online Appendix for "Timescale Betas and the Cross Section of Equity Returns: Framework, Application, and Implications for Interpreting the Fama–French Factors"

This Online Appendix consists of two sections. Section 1 contains details on the bootstrap methods used in the study. Section 2 provides figures and tables referenced in the main paper, but omitted for brevity.

1 Details of the Bootstrap Procedure

Our bootstrap procedure is based on the stationary bootstrap of Politis and Romano (1994). The stationary bootstrap is a block bootstrap with block lengths distributed as a geometric random variable. To determine the optimal average block length, we use the algorithm of Politis and White (2004), corrected by Patton et al. (2009). Specifically, for a given set of data, we apply the Politis–White algorithm to each series, each squared series, and the product of each pair of series and select the largest of these lengths as the average block length for the given data set.[1] We use 5,000 bootstrap replications in all cases, unless otherwise stated.

Shanken (1992) and Jagannathan and Wang (1998) show that the standard errors of the cross-sectional slopes must be adjusted to account for the estimation error in the factor loadings. Our empirical procedure adds another layer of estimation error, that is, error in the estimation of details (smooths) or wavelet (scaling) coefficients, which are used in the first-stage time-series regressions. We incorporate this additional sampling uncertainty by conducting wavelet decompositions (as well as the subsequent time-series and cross-sectional regressions) on resampled data, rather than resampling the decomposed data. In the process, our use of the stationary bootstrap also ensures that we do not impose time-series independence, so the obtained standard errors are robust to the presence of heteroskedasticity and autocorrelation.

In obtaining 95% confidence intervals for true cross-sectional R^2s, we follow Lewellen et al. (2010). Specifically, we simulate the sample distribution of the adjusted R^2 for a given true R^2, and plot the 2.5th and 97.5th percentiles (y-axis) against the corresponding true R^2 (x-axis). Repeating this for all values of the true R^2s ranging from 0.00 to 1.00 generates a graph similar to Figure 5 in Lewellen et al. (2010). We can then find a confidence interval for the true R^2, given a sample adjusted R^2, by slicing the graph along the y-axis (fixing y and then scanning across). To simulate the sample distribution of the adjusted R^2, for a given true R^2, we set the true factor loadings equal to the historical estimates while changing the vector of true expected returns to give the desired true R^2 (see footnote 6 in Lewellen et al. (2010) for details). The graph is based on 10,000 bootstrap simulations for each assumed true R^2 (1,000 sets of random expected returns, each with 10 bootstrap resamplings).

[1] For example, for the set of the FF three factors and the 25 size and book-to-market sorted portfolios, the average block length is selected to be 29.21.

The null distributions (and hence the p-values) for the WSSPE statistics are obtained with a bootstrap procedure similar to the one that produces the standard errors of the cross-sectional slopes. The only difference is that the test-asset returns are adjusted to be consistent with the pricing model before the data are resampled, which can be done by subtracting the pricing error for each test asset from the excess return series of the corresponding test asset (i.e., $R_{i,t} - \hat{\alpha}_i$ for each i and t). The empirical distributions for the F-statistics used in Table 4 are obtained in a similar way by adjusting the test-asset returns to be consistent with the corresponding null hypothesis before the random samples are generated; the null of $\boldsymbol{\lambda_1} = \mathbf{0}$, for example, can be imposed by subtracting $\hat{\boldsymbol{\lambda}}_1' \, \hat{\boldsymbol{\beta}}_{i,\bullet}$ (obtained from estimating (14)) from the excess return series of the corresponding test asset (i.e., $R_{i,t} - \hat{\boldsymbol{\lambda}}_1' \, \hat{\boldsymbol{\beta}}_{i,\bullet}$ for each i and t).

Finally, to obtain the 5% critical values in Figures 6 and 7, we resample the FF factors and state variables separately, rather than resample the vector of the FF factors and state variables. By doing so, we ensure that the null hypothesis of zero correlation is satisfied, while not changing the univariate distribution of any variable. In addition, by resampling the FF factors (rather than their details or smooths) and state variables (rather than their changes or residuals), we also incorporate the impact of estimation error on the null distributions of the partial correlations.

References

See references list in the main paper.

Patton, A., Politis, D.N., White, H., 2009. Correction to "Automatic block-length selection for the dependent boostrap" by D. Politis and H. White. Econometric Reveiws 28, 372–375.

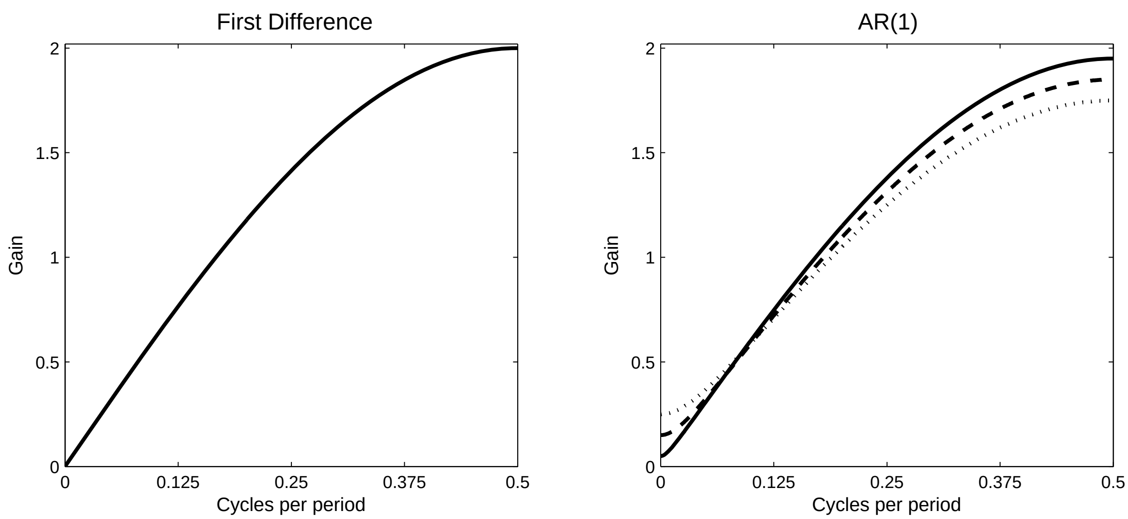
Politis, D.N., White, H., 2004. Automatic block-length selection for the dependent bootstrap. Econometric Reviews 23, 53–70.

Shanken, J., 1992. On the estimation of beta-pricing models. Review of Financial Studies 5, 1–33.

2 Additional Figures and Tables

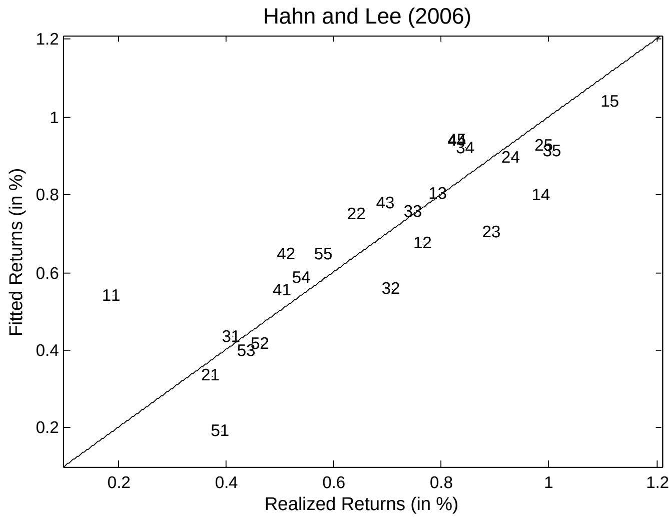
Figure A1: Gain Functions for First Difference and AR(1) Filters
Figure A2: Fitted versus Realized Returns
Table A1: ANOVA/ANCOVA-Based Cross-Sectional Regressions
Table A2: Additional MRA-Based Analyses
Table A3: Using Petkova's (2006) Innovations Factors
Table A4: MRA-Based Cross-Sectional Regressions: CCAPM

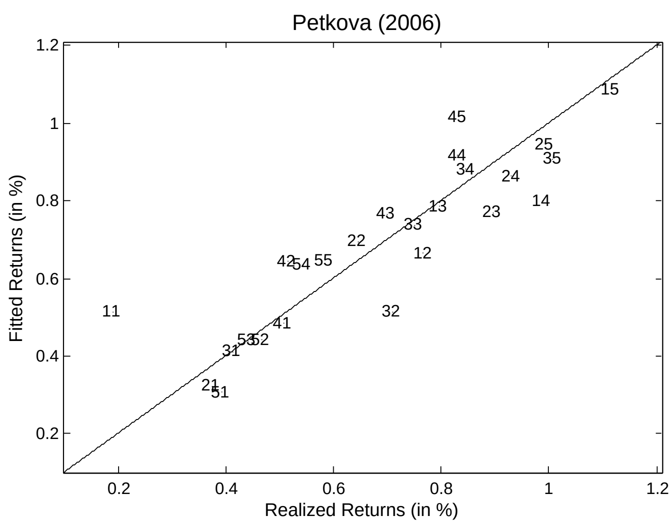
Figure A1
Gain Functions for First Difference and AR(1) Filters



This figure shows the gain functions for the first difference and AR(1) filters. In the right panel, the solid line, dashed line, and dotted line refer to the cases where the AR coefficient is 0.95, 0.85, and 0.75, respectively.

Figure A2
Fitted versus Realized Returns





This figure shows the pricing errors for each of the 25 size and book-to-market sorted portfolios of Hahn and Lee's (2006) and Petkova's (2006) models. Each two-digit number represents one portfolio. The first digit refers to the size quintile (1 being the smallest and 5 the largest), while the second digit refers to the book-to-market quintile (1 being the lowest and 5 the highest). The pricing errors are from the Fama–MacBeth regressions, similar to those in Table 1. Hahn and Lee's (2006) model is a three-factor model in which the factors are the excess market return, changes in term spread, and changes in the default spread. Petkova's (2006) model is a five-factor model in which the factors are the excess market return and innovations in the dividend yield, term spread, default spread, and short-term T-bill.

Table A1

ANOVA/ANCOVA-Based Cross-Sectional Regressions

	$E(R_{i,t}) = \lambda_m \beta^m_{i,\bullet} + \lambda_h \beta^{HML}_{i,\bullet} + \lambda_s \beta^{SMB}_{i,\bullet}$							FF
	$\bullet = w_1$	$\bullet = w_2$	$\bullet = w_3$	$\bullet = w_4$	$\bullet = w_5$	$\bullet = w_6$	$\bullet = v_6$	
Adj. R^2	0.55	0.60	0.69	0.63	0.55	0.56	0.58	0.62
	[0.40, 1.00]	[0.43, 1.00]	[0.54, 1.00]	[0.49, 1.00]	[0.48, 1.00]	[0.58, 1.00]	[0.65, 1.00]	[0.44, 1.00]
WSSPE	0.015	0.013	0.010	0.013	0.015	0.016	0.019	0.013
	[0.012]	[0.027]	[0.128]	[0.106]	[0.205]	[0.488]	[0.645]	[0.013]

This table compares the ability of models to explain the excess returns on 25 portfolios sorted by size and book-to-market ratio. The first seven columns correspond to seven variants of the FF model; the last column corresponds to the original FF model. The first two rows report the 95% confidence intervals for the true R^2s (in brackets), given the sample adjusted R^2s reported above them. The last two rows report the weighted sum of squared pricing errors (WSSPE) employed by Campbell and Vuolteenaho (2004) and the corresponding p-values (in brackets) for the null hypothesis that the pricing errors are jointly zero. The pricing errors in each model are computed under the restriction that the cross-sectional slope associated with a factor is equal to the factor's time-series average.

Table A2
Additional MRA-Based Analyses

	$E(R_{i,t}) = \lambda_m \beta^m_{i,\bullet} + \lambda_h \beta^{HML}_{i,\bullet} + \lambda_s \beta^{SMB}_{i,\bullet}$							FF
	$\bullet = d_1$	$\bullet = d_2$	$\bullet = d_3$	$\bullet = d_4$	$\bullet = d_5$	$\bullet = d_6$	$\bullet = s_6$	
Panel A: D(4) Filter in the Zeroth-Stage Wavelet Decompositions								
λ_m	0.298	0.247	0.225	0.177	0.210	0.253	0.337	0.411
λ_h	0.384	0.245	0.246	0.198	0.247	0.211	0.143	0.474
λ_s	0.171	0.130	0.122	0.095	0.111	0.090	0.129	0.219
Adj. R^2	0.587	0.630	0.712	0.664	0.634	0.618	0.632	0.645
WSSPE	0.015	0.014	0.011	0.013	0.013	0.015	0.018	0.013
Panel B: C(6) Filter in the Zeroth-Stage Wavelet Decompositions								
λ_m	0.299	0.249	0.229	0.180	0.213	0.258	0.341	0.411
λ_h	0.385	0.248	0.250	0.201	0.251	0.213	0.143	0.474
λ_s	0.172	0.132	0.123	0.097	0.114	0.092	0.132	0.219
Adj. R^2	0.587	0.629	0.713	0.664	0.634	0.616	0.631	0.645
WSSPE	0.015	0.014	0.011	0.013	0.013	0.015	0.018	0.013
Panel C: Rolling Windows in the First-Stage Time-Series Regressions								
λ_m	0.356	0.302	0.286	0.273	0.241	0.259	0.137	0.417
λ_h	0.372	0.303	0.267	0.189	0.219	0.137	0.112	0.462
λ_s	0.185	0.159	0.126	0.129	0.129	0.072	0.036	0.206
Adj. R^2	0.505	0.546	0.700	0.590	0.493	0.562	0.472	0.582
WSSPE	0.018	0.016	0.010	0.014	0.017	0.016	0.020	0.015
Panel D: Extending Windows in the First-Stage Time-Series Regressions								
λ_m	0.369	0.288	0.322	0.295	0.289	0.208	0.444	0.440
λ_h	0.394	0.298	0.293	0.259	0.241	0.177	0.109	0.457
λ_s	0.157	0.136	0.124	0.116	0.069	0.071	0.127	0.169
Adj. R^2	0.435	0.508	0.632	0.552	0.349	0.587	0.287	0.502
WSSPE	0.020	0.017	0.013	0.018	0.023	0.015	0.028	0.018
Panel E: Rebalancing *HML*, *SMB*, and Test Portfolios Every Five Years								
λ_m	0.376	0.322	0.284	0.279	0.312	0.366	0.445	0.460
λ_h	0.276	0.213	0.194	0.169	0.219	0.136	0.133	0.320
λ_s	0.179	0.147	0.139	0.135	0.131	0.128	0.214	0.214
Adj. R^2	0.523	0.562	0.683	0.618	0.536	0.441	0.352	0.605
WSSPE	0.010	0.009	0.005	0.007	0.010	0.015	0.017	0.008
Panel F: July 1963 to December 1991								
λ_m	0.331	0.285	0.319	0.234	0.368	0.181	0.425	0.417
λ_h	0.387	0.299	0.299	0.271	0.159	0.248	0.241	0.451
λ_s	0.204	0.182	0.150	0.122	0.167	0.168	0.268	0.223
Adj. R^2	0.619	0.672	0.743	0.640	0.646	0.640	0.437	0.671
WSSPE	0.014	0.013	0.009	0.014	0.012	0.013	0.024	0.012
Panel G: January 1992 to June 2008								
λ_m	0.348	0.346	0.235	0.294	0.160	0.555	0.404	0.417
λ_h	0.400	0.275	0.314	0.231	0.436	0.175	0.367	0.520
λ_s	0.193	0.153	0.144	0.168	0.133	0.018	-0.015	0.218
Adj. R^2	0.226	0.269	0.462	0.436	0.325	0.179	0.139	0.397
WSSPE	0.046	0.044	0.031	0.030	0.036	0.058	0.075	0.035

(continued on next page)

Table A2 (continued)
Additional MRA-Based Analyses

	$E(R_{i,t}) = \lambda_m \beta_{i,\bullet}^{m} + \lambda_h \beta_{i,\bullet}^{HML} + \lambda_s \beta_{i,\bullet}^{SMB}$							FF
	$\bullet = d_1$	$\bullet = d_2$	$\bullet = d_3$	$\bullet = d_4$	$\bullet = d_5$	$\bullet = d_6$	$\bullet = s_6$	
Panel H: Augmenting the Test Assets with Industry, CAPM Beta, and Cluster Portfolios								
λ_m	0.384	0.324	0.305	0.272	0.322	0.346	0.394	0.462
λ_h	0.287	0.256	0.285	0.145	0.165	0.129	0.044	0.370
λ_s	0.168	0.135	0.134	0.126	0.143	0.137	0.203	0.190
Adj. R^2	0.270	0.301	0.366	0.238	0.191	-0.467	-0.400	0.325
WSSPE	0.045	0.047	0.043	0.046	0.054	0.104	0.099	0.044
Panel I: Excluding Observations Heavily Affected by Circularity								
λ_m	0.336	0.287	0.317	0.228	0.365	0.175	0.389	0.417
λ_h	0.387	0.299	0.313	0.261	0.200	0.300	0.139	0.451
λ_s	0.202	0.186	0.155	0.108	0.163	0.160	0.156	0.223
Adj. R^2	0.614	0.670	0.746	0.621	0.612	0.663	0.475	0.671
WSSPE	0.014	0.013	0.010	0.015	0.013	0.012	0.022	0.012

Panels A and B contain results when the Daubechies *extremal phase* filter of width $L = 4$ (denoted by D(4)) and *coiflet* filter of width $L = 6$ (denoted by C(6)) are used, respectively. Panels C and D contain results when the factor loadings are estimated over 120-month rolling windows and extending windows, respectively. Panel E contains results when *HML*, *SMB*, and the 25 size and book-to-market portfolios are rebalanced every five years. Panels F and G contain results when we examine separately the periods before and after January 1992, respectively. Panel H contains results when we augment the test assets with industry, CAPM beta, and the cluster portfolios of Ahn et al. (2009). Ten industry portfolios are from Kenneth French's Web site. Ten cluster portfolios are from Robert Dittmar's Web site. Panel I contains results when we use data from January 1947 to conduct the MRA and discard the first and last 198 observations in the resulting series. The first seven columns correspond to seven variants of the FF model; the last column corresponds to the original FF model. The first three rows in each panel report slope estimates; the last two rows report adjusted R^2s and weighted sum of squared pricing errors (WSSPE) employed by Campbell and Vuolteenaho (2004).

Table A3
Using Petkova's (2006) Innovations Factors

Dep. Var.	Independent Variables						
	Const.	R_m	u^{div}	u^{term}	u^{def}	u^{rf}	Adj. R^2
Panel A: Decomposing the Regression of *HML* on the Independent Variables							
HML	0.61	-0.27	-0.06	0.06	0.02	-0.00	0.18
	(5.64)	(-4.48)	(-1.72)	(1.22)	(0.78)	(-0.13)	
HML^{d_1}	0.08	-0.12	-0.03	0.02	0.04	-0.01	0.10
	(2.95)	(-3.84)	(-1.09)	(0.77)	(2.11)	(-0.17)	
HML^{d_2}	0.04	-0.07	-0.01	-0.01	0.01	-0.01	0.06
	(2.75)	(-4.36)	(-0.79)	(-0.56)	(0.35)	(-0.81)	
HML^{d_3}	0.03	-0.04	-0.01	0.04	-0.01	0.00	0.05
	(2.78)	(-3.78)	(-0.81)	(3.16)	(-0.86)	(0.28)	
HML^{d_4}	0.02	-0.02	-0.01	0.01	-0.00	0.00	0.02
	(2.80)	(-2.81)	(-0.89)	(0.71)	(-0.52)	(0.67)	
HML^{d_5}	0.02	-0.02	-0.02	0.00	-0.01	0.01	0.02
	(5.43)	(-2.46)	(-2.46)	(0.43)	(-1.36)	(1.25)	
HML^{d_6}	-0.00	-0.00	0.00	0.01	-0.00	0.00	0.00
	(-0.00)	(-1.13)	(0.50)	(1.50)	(-0.46)	(0.21)	
HML^{s_6}	0.42	-0.00	0.01	-0.00	0.00	-0.00	0.01
	(3.72)	(-0.88)	(1.49)	(-0.25)	(0.13)	(-0.37)	
Panel B: Decomposing the Regression of *SMB* on the Independent Variables							
SMB	0.09	0.22	0.06	0.02	-0.07	-0.04	0.10
	(0.58)	(5.30)	(1.41)	(0.63)	(-2.16)	(-1.30)	
SMB^{d_1}	-0.01	0.04	-0.01	-0.04	-0.00	-0.04	0.01
	(-0.71)	(1.83)	(-0.41)	(-1.74)	(-0.19)	(-1.64)	
SMB^{d_2}	-0.06	0.08	0.02	0.03	-0.03	-0.00	0.08
	(-4.29)	(6.03)	(1.37)	(2.04)	(-1.78)	(-0.03)	
SMB^{d_3}	-0.04	0.06	0.01	0.01	-0.02	0.00	0.09
	(-3.57)	(7.32)	(1.15)	(1.36)	(-2.84)	(0.38)	
SMB^{d_4}	-0.02	0.03	0.01	0.00	-0.01	0.00	0.04
	(-3.26)	(4.64)	(0.66)	(0.62)	(-1.80)	(0.75)	
SMB^{d_5}	-0.02	0.01	0.01	-0.00	0.01	-0.00	0.02
	(-4.96)	(2.16)	(1.98)	(-0.01)	(1.11)	(-0.68)	
SMB^{d_6}	-0.00	-0.00	0.01	0.01	0.00	-0.00	-0.00
	(-1.22)	(-0.58)	(1.04)	(1.19)	(0.26)	(-0.43)	
SMB^{s_6}	0.23	-0.00	0.01	-0.00	-0.00	-0.01	-0.00
	(1.49)	(-0.31)	(0.91)	(-0.29)	(-0.53)	(-0.88)	

Panel A decomposes the time-series regressions of *HML* on the term factor and the default factor, controlling for market excess returns and other state variable risk proxies considered in Petkova (2006). Panel B decomposes the time-series regressions of *SMB* on the same set of independent variables. The numbers reported are the coefficient estimates of the regressions with the associated t-statistics in parentheses. The t-statistics are obtained from a bootstrap procedure designed to account for time-series dependence, as well as estimation error in the dependent variables and independent variables. The last column reports adjusted R^2s. The independent variables include residuals in dividend yield, term spread, default spread, and short-term rate, estimated from a vector autoregressive (VAR) model. The dividend yield is defined as the dividend yield of the CRSP value-weighted portfolio (computed as the sum of dividends over the last 12 months, divided by the level of the index). The short-term rate is defined as the 1-month T-bill rate, obtained from the Ibbotson Associates.

Table A4
MRA-Based Cross-Sectional Regressions: CCAPM

Panel A: $E(R_{i,t}) = \lambda_c \beta^c_{i,\bullet}$			
	λ_c	Adj. R^2	WSSPE
		$\bullet = d_1$	
Estimate	0.42	-3.63	0.576
t-value	2.54		[0.099]
Bootstrap-t	1.57		
		$\bullet = d_2$	
Estimate	0.15	-0.06	0.088
t-value	2.90		[0.260]
Bootstrap-t	1.27		
		$\bullet = d_3$	
Estimate	0.22	-0.12	0.117
t-value	2.93		[0.419]
Bootstrap-t	1.32		
		$\bullet = d_4$	
Estimate	0.34	-0.14	0.133
t-value	2.98		[0.635]
Bootstrap-t	1.53		
		$\bullet = s_4$	
Estimate	0.92	-1.21	0.225
t-value	3.19		[0.853]
Bootstrap-t	2.33		

Panel B: $E(R_{i,t}) = \lambda_c \beta^c_i$			
	λ_c	Adj. R^2	WSSPE
Estimate	0.49	-0.31	0.113
t-value	2.88		[0.211]
Bootstrap-t	2.02		

This table reports the cross-sectional regression results using the excess returns on 25 portfolios sorted by size and book-to-market ratio. Panel A contains results for five variants of the CCAPM; Panel B contains results for the original CCAPM. The slope estimates are expressed as percentage per quarter. The first set of t-statistics stands for the Fama–MacBeth estimate. The second set, indicated by Bootstrap-t, is obtained from a bootstrap procedure designed to account for time-series dependence, as well as estimation error in details, smooths, and factor loadings. The second column reports the adjusted cross-sectional R^2s. The last column reports the weighted sum of squared pricing errors (WSSPE) employed by Campbell and Vuolteenaho (2004) and the corresponding p-values (in brackets) for the null hypothesis that the pricing errors are jointly zero.