A Closed-Form Solution to a Dynamic Portfolio Optimization Problem

Zhong-Fei Li
Department of Finance, Lingnan (University) College, Sun Yat-Sen University
Guangzhou 510275, People's Republic of China

Hailiang Yang
Department of Statistics and Actuarial Science, The University of Hong Kong
Pokfulam Road, Hong Kong

Abstract
The pioneering work of Markowitz (1952) introduced the mean-variance framework for portfolio selection and risk management which are important problems in investment finance. The mean-variance approach became the foundation of modern finance theory and inspired literally a substantial number of extensions and applications. Great progresses in this field have been made in different directions, such as (1) extending to multi-period or continuous-time settings (see, e.g., Li and Ng (2000), Merton (1990), Samuelson (1969), and Zhou and Li (2000)), and (2) modeling more reasonable risk measure (see, e.g, Basak and Shapiro (2001), and Emmer, Klüppelberg and Korn (2001)).

Recently, there have been works on constant-rebalanced portfolio (CRP) investment strategies for many portfolio optimization problems, see, for example, Cover (1991), Emmer, Klüppelberg and Korn (2001), and Helmbold et al (1998). A constant-rebalanced portfolio strategy is an investment strategy which keeps the same distribution of wealth among a set of securities from time to time (or from period to period).

In this paper we consider a continuous-time Markowitz mean-variance type portfolio optimization problem where the variance is replaced by a Earnings-at-Risk (EaR) of terminal wealth. In a Black-Scholes setting, we obtain closed-form expressions for best constant-rebalanced portfolio investment strategies and the mean-EaR efficient frontier.

Keywords
Dynamic portfolio selection; Earnings-at-Risk; constant-rebalanced portfolios; Black-Scholes model

---

This research is supported by a grant of the National Natural Science Foundation of China (No. 10171115), a "Tenth Five-Year Plan" project of Ministry of Education of China (No. 01JA630009), a grant of the Natural Science Foundation of Guangdong Province (No. 011193), and a research grant of Hong Kong Research Grant Council (HKU 7139/01H).

Corresponding Author. E-mail: lnslzf@zsu.edu.cn.