The Hong Kong Polytechnic University
Department of Applied Mathematics

Colloquium

On

From the Bergman kernel to holomorphic isometries: a method of analytic continuation

by

Professor Mok Ngaiming
Department of Mathematics, The Hong Kong University

Abstract

Bounded domains in the complex Euclidean space $\mathbb{C}^n$ are among the first objects of study in the theory of functions of several complex variables. In the pioneering work of Bergman in the early twentieth century, special metrics are introduced into the study of such domains as tools for the study of holomorphic functions and holomorphic maps defined on such domains. These metrics were later called Bergman metrics. They are Kähler metrics which are preserved under bijective holomorphic maps between bounded domains. In the special case of the disk the Bergman metric is nothing other than the Poincaré metric, and for bounded domains homogeneous under the group of biholomorphisms, e.g., bounded symmetric domains, the Bergman metric is of constant Ricci curvature, i.e., the Kähler-Einstein metric up to a normalizing constant.

Bergman metrics arise from Bergman kernels, which are defined analytically in terms of square-integrable holomorphic functions. Since biholomorphic maps are isometries with respect to the Bergman metric, the boundary behavior of such maps has been studied by means of the Bergman kernel and the Bergman metric, as exemplified by the seminal work of C. Fefferman on biholomorphic maps between strictly pseudoconvex bounded domains. In the non-equidimensional case one is naturally led to study holomorphic isometries. Motivated by a problem from Arithmetic Geometry raised by Clozel-Ullmo, we study the question of characterizing germs of holomorphic isometric immersions between bounded domains with respect to the Bergman metric.

Extension and rigidity problems for holomorphic isometries into possibly infinite-dimensional space forms dated back to works of Bochner and Calabi. For a bounded domain D equipped with the Bergman kernel $K_D(z,w)$, the function $\log K_D(z, z)$ serves as a potential function for the Bergman metric ds^2_D, and the choice of an orthonormal basis for the Hilbert space $H^2(D)$ of square-integrable holomorphic functions defines a holomorphic isometric embedding of $(D; ds^2_D)$ into the infinite-dimensional projective space $\mathbb{P}^\infty$ equipped with the Fubini-Study metric. In the simply connected case interior extension results already follow from Calabi's seminal work in 1953 on the subject. Here we are primarily concerned with extension beyond the boundary for bounded domains with specific realizations, notably bounded symmetric domains in their Harish-Chandra realizations. The upshot is that the graph of a germ of holomorphic isometry extends algebraically in the latter case. On the other hand, we have found examples of proper holomorphic isometric embeddings of the Poincaré disk into bounded symmetric domains which are not totally geodesic, giving in particular counter-examples to a conjecture of Clozel-Ullmo's.

Date : 9 April, 2009 (Thursday)
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