THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

DEPARTMENT OF APPLIED MATHEMATICS
應 用 數 學 系

Dr. LI Buyang

Research interest :

- **Numerical Analysis**

Start-up grant project

Convergence of fully discrete finite element solutions of nonlinear parabolic equations in nonsmooth domains

Abstract

We investigate stability and convergence of fully discrete finite element solutions of some nonlinear parabolic partial differential equations (PDEs) from physics and engineering, such as time-dependent Ginzburg-Landau equations and the L^2-gradient flow PDEs, in nonsmooth domains such as nonconvex polygons and nonconvex polyhedra. For many such problems, the nonlinearities may grow without bound as the solution tends to ∞. The diffusion coefficients may also depend on the solution and tend to 0 or $+\infty$ as the solution tends to ∞. The main difficulty in numerical analysis of such nonlinear problems is to control the numerical solutions in certain L^p or $W^{1,p}$ norms in order to control the nonlinearities. Due to this difficulty, many existing works require some stability conditions which relate the time step size to the spatial mesh size, such as $\tau=O(h^\alpha)$, which help to control the nonlinear terms by using inverse inequalities of the finite element spaces. Such stability conditions are not convenient in practical computations. The objective of this project is to remove such stability conditions in error analysis and clear up the misgivings for the time step size in practical computations. This has been done for some specific nonlinear parabolic PDEs in smooth domains where the solutions are sufficiently smooth, by using the error splitting technique developed recently in the L^2-norm setting. We shall further investigate the stability and convergence of numerical solutions of nonlinear parabolic PDEs in nonconvex polygons and polyhedra, where the regularity of the solutions are very weak. For some PDE problems, such as the time-dependent Ginzburg-Landau equations, the solutions are not even in $H^1(\Omega)$ in such nonsmooth domains.

For PDE problems which admit strong solutions, we investigate convergence rates of numerical solutions subject to quasi-uniform meshes and locally refined meshes, by extending the error splitting technique to the L^p-norm and $W^{1,p}$-norm setting with high-order time discretization methods. For PDE problems which only admit weak solutions, we shall propose numerical methods which are compatible with the structure of the PDEs and the regularity of the solutions, and prove convergence of the numerical solutions. In both cases, we shall avoid imposing such stability conditions as $\tau=O(h^\alpha)$, by extending the error splitting technique to the L^p-norm and $W^{1,p}$-norm setting. The techniques developed in this project can be applied to more general nonlinear parabolic PDE problems.