Lagrange-type functions in constrained optimization

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Classical Lagrange and penalty functions can be efficiently applied only to solving some special classes of constrained optimization problems. Thus there is a clear need to introduce and study more general functions (we shall call them *Lagrange-type functions*), which can efficiently reduce broad classes of constrained problems to unconstrained optimization. An augmented Lagrangian is one of the well-known examples of Lagrange-type functions. Different types of Lagrange-type functions also can be studied and applied . A Lagrange-type function allows one to formulate a dual problem.

We shall discuss the following questions:

When does the weak duality hold, that is the value of the dual problem does not exceed the value of the primal problem;
When does the zero duality gap property hold, that is the values of both problems coincides;
When does an exact Lagrange parameter exist (the zero duality gap property holds and the dual problem has a solution);
When does a strong exact parameter exist (the set of solution of the unconstrained problem with the exact parameter coincides with the set of solution of the given (primal) problem).

Penalty-type functions form an important subclass of Lagrange-type functions. Assume that penalty parameters are vectors with nonnegative coordinates. Then under natural assumptions each vector, which is greater than a vector of exact parameters, is also a vector of exact parameter. However, very large exact parameters lead to ill-conditioned unconstrained problems. Thus the following question arises:

How to construct a penalty-type function, which possesses a fairly small (strong) exact penalty parameter?

We propose a new penalty-type function with a small penalty parameter. Numerical experiments confirm that this function can be efficiently applied for solving some non-convex problem, including concave optimization.