CONTROLLABILITY OF REDUCED SYSTEMS

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Abstract

The results presented below were obtained jointly with P. Birtea and M. Puta. They generalize a Lie-Poisson control theorem of Manikonda and Krishnaprasad to arbitrary symmetry quotients of symplecitc manifolds.

Let (M, ω) be a symplectic manifold which is the phase space of classical conservative mechanical system. Consider the control system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i, \ x \in M, \ u = (u_1, ... u_m) : \mathsf{R} \to \mathsf{B} \subset \mathsf{R}^m,$$

where

$$f(x) = \sum_{i=1}^{n} \{x_i, H\}_\omega \frac{\partial}{\partial x_i}$$

is a Hamiltonian vector field on M, $\{\cdot, \cdot\}_\omega$ is the Poisson bracket given by the symplectic form ω, $n = \dim M$, B is a bounded set, and u is measurable.

Assume that the Lie group G acts freely and properly on M preserving the symplectic form ω and the vector fields f and g_i. Then the reduced dynamics on M/G is given by

$$\dot{\tilde{x}} = \tilde{f}(\tilde{x}) + \sum_{i=1}^{m} \tilde{g}_i(\tilde{x}) u_i,$$

where the reduced vector field $\tilde{f}$ is also Hamiltonian on the reduced Poisson manifold $\left(M/G, \{\cdot, \cdot\}_{M/G}\right)$.

Sufficient conditions for the controllability of the reduced system will be presented. This will be done by finding topological conditions under which the well known sufficient conditions

(i) $\tilde{f}$ is weakly positively Poisson stable (WPPS)
(ii) the Lie algebra rank condition (LARC) is satisfied

for the controllability of the reduced system are satisfied. The result will be applied to the motion of three point vortices in the plane, the three-wave interaction, and two coupled planar rigid bodies.