## A Closed-Form Solution to a Dynamic Portfolio Optimization Problem<sup>\*</sup>

Zhong-Fei Li<sup>y</sup>

Department of Finance, Lingnan (University) College, Sun Yat-Sen University Guangzhou 510275, People's Republic of China Hailiang Yang

Department of Statistics and Actuarial Science, The University of Hong Kong Pokfulam Road, Hong Kong

## Abstract

The pioneering work of Markowitz (1952) introduced the mean-variance framework for portfolio selection and risk management which are important problems in investment <sup>-</sup>nance. The mean-variance approach became the foundation of modern <sup>-</sup>nance theory and inspired literally a substantial number of extensions and applications. Great progresses in this <sup>-</sup>eld have been made in di<sup>®</sup>erent directions, such as (1) extending to multi-period or continuous-time settings (see, e.g., Li and Ng (2000), Merton (1990), Samuelson (1969), and Zhou and Li (2000)), and (2) modeling more reasonable risk measure (see, e.g, Basak and Shapiro (2001), and Emmer, Kläppelberg and Korn (2001)).

Recently, there have been works on constant-rebalanced portfolio (CRP) investment strategies for many portfolio optimization problems, see, for example, Cover (1991), Emmer, Kläppelberg and Korn (2001), and Helmbold et al (1998). A constant-rebalanced portfolio strategy is an investment strategy which keeps the same distribution of wealth among a set of securities from time to time (or from period to period).

In this paper we consider a continuous-time Markowitz mean-variance type portfolio optimization problem where the variance is replaced by a Earnings-at-Risk (EaR) of terminal wealth. In a Black-Scholes setting, we obtain closed-form expressions for best constant-rebalanced portfolio investment strategies and the mean-EaR  $e\pm$ cient frontier.

## Keywords

Dynamic portfolio selection; Earnings-at-Risk; constant-rebalanced portfolios; Black-Scholes model

<sup>&</sup>lt;sup>x</sup>This research is supported by a grant of the National Natural Science Foundation of China (No. 10171115), a \Tenth Five-Year Plan" project of Ministry of Education of China (No. 01JA630009), a grant of the Natural Science Foundation of Guangdong Province (No. 011193), and a research grant of Hong Kong Research Grant Council (HKU 7139/01H).

<sup>&</sup>lt;sup>y</sup>Corresponding Author. E-mail: Inslzf@zsu.edu.cn.