Jobshop Scheduling with Variable Duration and Multiple Resources

Yihua LI, ILOG in UK

Jobshop scheduling is to find a set of task starting times on each machine to minimize total make span. There are two types of constraints involved in the problem: one is task sequence requirement within each job; the other is a limit of machine resources (each machine can only processes one task each time). The second type of constraint is non-linear, which leads the problem difficult to be solved optimally within a reasonable computing time.

I find a way to convert those non-linear constraints into linear ones by introducing a set of boolean variable and convert the problem to a linear mixed integer-programming problem. I also design a specific solution method to solve the problem optimally and quickly. For a problem with 12 jobs, each job having 12 tasks on 12 machines, I solved it in 15 seconds comparing with 38 minutes by Scheduler/Solver and 45 minute by CPLEX.

Now we are dealing with a problem with variable durations on multiple machines, and try to solve it optimally and efficiently. A mathematic model is formulated as follows:

Minimize $\quad z \qquad$ (makespan) $\qquad$ (1.0)

subject to:

$$z \geq x_{in} + d_{in}, \quad \forall i \in I \quad \text{(Minimize max makespan)} \qquad (1.1)$$

$$x_{i,j+1} \geq x_{ij} + d_{ij}, \quad \forall_i \in I, \forall_j \in J_i \quad \text{(Task sequence)} \qquad (1.2)$$

$$y_{ij} = x_{is}, \quad \forall_i \in I, \forall_j \in S_i \quad \text{(Supper task start)} \qquad (1.3)$$

$$y_{ij} + t_{ie} = x_{ie} + d_{ie}, \quad \forall_i \in I, \forall_j \in S_i \quad \text{(Supper task end)} \qquad (1.4)$$

$$y_{ij} - y_{i'j'} - t_{i'j'} \geq M(b_{ij} - 1), \quad \forall_{ij} \in R \quad \text{(resource limit)} \qquad (1.5)$$

$$y_{i'j'} - y_{ij} - t_{ij} \geq -M\, b_{ij}, \quad \forall_{ij} \in R \quad \text{(resource limit)} \qquad (1.6)$$

$$y_{ij} - y_{i'j'} - t_{i'j'} \geq M(B_{ij} - 1), \quad \forall_{ij} \in A \quad \text{(resource limit)} \qquad (1.7)$$

$$y_{i'j'} - y_{ij} - t_{ij} \geq -M\, B_{ij}, \quad \forall_{ij} \in A \quad \text{(resource limit)} \qquad (1.8)$$

$$\sum_i \sum_j B_{ij} \leq 1 \quad \text{(Arm resource)} \qquad (1.9)$$

$$all \quad x_{ij}, y_{ij}, \geq 0, \text{integer}; \quad b_{ij}, B_{ij} = 0, 1 \qquad (2.0)$$

where:

A — a set of tasks using Arm resource,
d_{ij} — duration of basic task j in job i,
I — a set of job,
$\mathbf{J_i}$ — a set of basic tasks in job i,
R — a set of resources,
$\mathbf{S_i}$ — a set of supper tasks in job i,
M — big constant,
t_{ij} — duration of supper task j in job i,

b_{ij} — 1—task i is early than task j, 0--reversed,
B_{ij} — 1—task i and task j are overlapped on Arm resource , 0--otherwise,
x_{ij} — starting time of basic task j in job i,
y_{ij} — starting time of supper task j in job i,
z — makespan, maximum completion time for all jobs,

This is an Integer Linear Programming problem, which is NP-Completed hard. According to the special structure of the problem, we develop a heuristic to get an initial feasible solution quickly, and then call CPLEX to solve the problem optimally. Since this model is so general, it can deal with most of Jobshop scheduling problem in real world. That is, either there are constant or variable durations, either each task is processed by one or more machine(s) simultaneously, either one machine can process one or more task(s) simultaneously; an optimal solution can be obtained by a heuristic and CPLEX within a reasonable computing time. For a testing, a Wafer Chip Processing problem with 4 jobs and each job having 20 tasks over 9 machines (See Figure 1) has been solved optimally in less 30 seconds.

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MagnaTran 7

MagnaTran 7 is available in 2 axis (R, θ) and 3 axis (R, θ , Z) models, with 3 different patented arm configurations in various sizes, and with a variety of passive support end effectors, for handling up to 300mm size wafers or similar substrates.

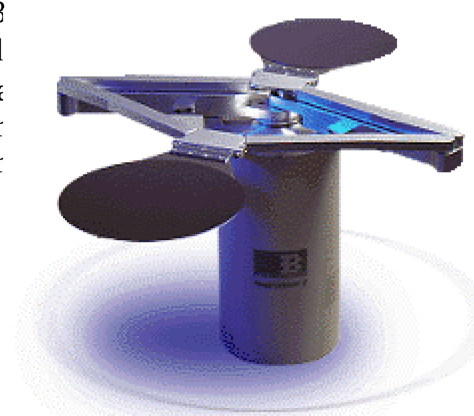

Figure 1. Brooks Automation

Table 1. Gantt Chart for processing task scheduling by **JView** of ILOG.