



**The Hong Kong Polytechnic University
Department of Applied Mathematics**

**Seminar
On**

**Convergence of Krylov Subspace Methods
for Least Squares Problems**

by

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Abstract

The standard iterative method for solving large sparse least squares problems $\min \|b - Ax\|_2$, where A is an m by n real matrix, is the CGLS method, or its stabilized version LSQR, which is mathematically equivalent to applying the conjugate gradient method to the normal equation $A^T Ax = A^T b$.

We consider alternative methods using an n by m matrix B and applying the Generalized Minimal Residual (GMRES) method to $\min \|b - ABz\|_2$ or $\min \|Bb - BAx\|_2$.

We give a sufficient condition concerning B for the GMRES methods to give a least squares solution without breakdown for arbitrary b , for over-determined, under-determined and possibly rank-deficient problems. We then give a convergence analysis of the GMRES methods as well as the CGLS method.

Then, we propose using the robust incomplete factorization (RIF) for B .

Finally, we analyze the effect of the condition number and distribution of singular values of A on the convergence of the methods by numerical experiments on over-determined problems. We also show that for ill-conditioned problems, the GMRES methods give least squares solutions faster than the CGLS and LSQR methods with RIF, and are similar in performance to the reorthogonalized CGLS with RIF.

Date : 12 December, 2008 (Friday)
Time : 2:00 – 3:00 p.m.
**Venue : Departmental Conference Room HJ610
The Hong Kong Polytechnic University**

***** ALL ARE WELCOME *****